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ELECTRIC RESISTANCE DRYING OF CERAMICS

Part I — Mathematical model

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The study deals with a procedure for investigation of electric resistance drying of ceramic bodies in plastic state, based on mathematical modelling. The mathematical model is resolved for constant material characteristics by the method of Laplace's transformation; the solution for material characteristics dependent on temperature and moisture content is based on the Crank—Nicolson network method.

INTRODUCTION

The technological operation of the drying of ceramic bodies represents a process having a significant effect on the final properties of ware. In dependence on the way the body is heated, it is possible to distinguish various types of drying (such as convective, electric resistance or high-frequency methods). In electric resistance drying, the ceramic body behaves as a conductor in which Joule heat is generated as a result of passage of electric current.

The study of electric resistance drying (further on ERD) can be based on two procedures. The first, an empirical one, is based on establishment of the relationship between the electric voltage, the body temperature attained and the mechanical properties of the green body, or its visual assessment after drying. The drying voltage is derived from the results of such tests. The procedure is experimentally demanding and time consuming and not always permits the correct conditions for drying to be found. The other method is based on mathematical modelling and can be schematically divided into the following stages:

1. Using basic balance and constitutive equations and the material functions established, a general model of the process is designed.
2. On the basis of an analysis of the given operation, the final and the boundary conditions for the resolving of the general model are determined.
3. Solution of the general model for the initial and boundary conditions yields a mathematical form of the dependence of a chosen parameter on the various variables.
4. As a number of initial assumptions and simplifying conditions has been accepted in the derivation of the model, a comparison of experimental results with those obtained from the model should prove that the assumptions and simplifying conditions introduced into the problem formulation are actually suitable.

The present paper has the purpose to illustrate a procedure employed in the study of ERD of ceramic bodies in plastic state.

BALANCE AND CONSTITUTIVE EQUATIONS

If the saturated ceramic mix is regarded as a binary mixture composed of incompressible components (ceramic material and electrolyte, i.e. water and soluble salts), then the transfer of the liquid component can be considered as diffusion in a binary mixture and the following equation used in describing the flux [1]:

$$\boldsymbol{h} = -D \operatorname{grad} C - D_T \operatorname{grad} T - D_P \operatorname{grad} P, \tag{1}$$

where $\boldsymbol{h}$ is the bulk flux density, D, D_T and D_P are the diffusion, thermodiffusion and barodiffusion coefficients respectively, T is temperature, P is pressure and C is the moisture content by volume. The heat transfer can be regarded as conduction of heat in a ceramic mix, and the flux described by the constitutive equation of heat conduction in the form

$$\boldsymbol{q} = -\lambda \operatorname{grad} T, \tag{2}$$

where $\boldsymbol{q}$ is the heat flux density, λ is the coefficient of thermal conductivity. The respective moisture content and heat balances are then given by the equations

$$\frac{\partial C}{\partial t} = -\operatorname{div} \boldsymbol{h}, \tag{3}$$

and

$$\varrho c_p \frac{\partial T}{\partial t} = -\operatorname{div} \boldsymbol{q} + r\varrho, \tag{4}$$

where ϱ and c_p are density and specific heat respectively, t is time and r is the specific heat source.

If the heat is generated by electric current, equations (1) and (2) have to be supplemented with the constitutive equation characterizing the electric properties of ceramic mix having the form [2]:

$$\boldsymbol{J} = -\sigma \operatorname{grad} \varphi - L_1 \operatorname{grad} T - L_2 \operatorname{grad} c, \tag{5}$$

where $\boldsymbol{J}$ is the current density, σ is electric conductance, φ is electric potential, L_1 is the thermoelectric coefficient and L_2 is the diffusion-electric coefficient, $\boldsymbol{c}$ is the concentration of soluble salts. If the last two terms in equation (5) can be neglected, the equation aquires the form for of Ohm's law,

$$\boldsymbol{J} = -\sigma \operatorname{grad} \varphi, \tag{6}$$

and the heat source in equation (4) can be considered as a volume source of heat described by the equation

$$\varrho r = RI^2/V, \tag{7}$$

where R is the resistivity, I is the current and V is the body volume.

Study [3—5] showed that the effect of the temperature gradient on water transfer in a saturated ceramic mix can be neglected. On introducing the effective diffusion coefficient, i.e. one including in its value also the effect of capillary barodiffusion,

$$D_{\text{ef}} = D + D_P \frac{\partial P}{\partial C} \tag{8}$$

equation (1) can be expressed in the form

$$\boldsymbol{h} = -D_{ef} \operatorname{grad} C. \tag{9}$$

Having joined equations (2) and (9) with equations (3), (4) and (7), the one-dimensional mass and heat balances can be expressed in the forms

$$\partial_t C = \partial_x (D_{ef}\, \partial_x C), \tag{10}$$

$$\varrho c_p\, \partial_t T = \partial_x (\lambda\, \partial_x T) + RI^2/V. \tag{11}$$

The one-dimensional equations (2) and (9) acquire thus the forms

$$\boldsymbol{h} = -D_{ef}\, \partial_x C \tag{12}$$

and

$$\boldsymbol{q} = -\lambda\, \partial_x T. \tag{13}$$

The set of equations (10—13) allows all technological operations associated with mass and heat transfer and a bulk heat source in a saturated ceramic mix to be modelled. The given operation can then be rendered concrete by introduction of initial and boundary conditions.

FORMULATION OF INITIAL AND BOUNDARY CONDITIONS

The course of ERD in a medium with constant parameters of the drying environment can be described by the time dependence of mean body temperature $\bar{T}$, the passing current I and the body weight G. Fig. 1 shows that the course of ERD can be divided into three characteristic periods:

I. The heating-through period;
II. The period of constant drying rate;
III. The period of decreasing drying rate.

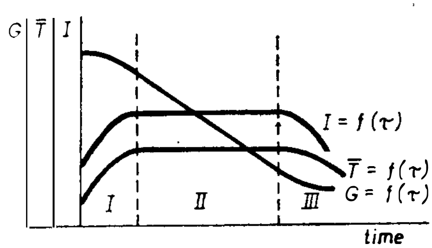

Fig. 1. Schematic diagram of the course of ERD in a ceramic body.

During period I, the passage of electric current and exchange of heat with the environment brings about heating up of the body, which results in the formation of temperature gradients in the body. The end of this period is characterized by attainment of the state when temperature $\bar{T}$ = const, current I = const and the weight is a liner function of time, $G = G(t)$. Period II of drying exhibits a constant mean body temperature, a constant current and a linear dependence of body weight on time. The end of the period is given by the so-called critical point, i.e. one at which a decrease of the passing current and of the mean body temperature occurs as a result of a decrease of the moisture content below its critical value.

In the course of drying in periods I and II, water evaporates from the body surface and passes to the ambient atmosphere [8]. The surface moisture flux is equalized by the flux of moisture from the body interior, and this leads to creation of moisture fields responsible for non-uniform shrinkage of the individual body portions. Stress fields arise in consequence of uneven shrinkage and if these exceed a maximum admissible value, deformation or failure of the green body takes place. To be able to describe the course of ERD, one must therefore know the time course of moisture and temperature fields in the body.

As there is a direct relationship between the drying rate and the surface flux of moisture, the surface flux of temperature and the amount of heat generated by the bulk heat source and the heating rate, the following initial and boundary conditions can be chosen for the first and the second period of ERD:

$$\begin{aligned} t = 0, \quad & x \in \langle 0, L \rangle, \quad T = T_0, \quad C = C_0, \\ & I = 0, \quad T = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} & t > 0, \quad x = 0, \quad \partial_x C = \partial_x T = 0, \\ & x = L, \quad \mathbf{h}_L = \mathbf{h}_L(t) = -D_{\mathrm{ef}}\, (\partial_x C)_L \\ & \mathbf{q}_L = \mathbf{q}_L(t) = -\lambda\, (\partial_x T)_L \\ & I = I(t), \quad U = U(t) \end{aligned} \tag{15}$$

where L is the body thickness, C_0 and T_0 are the initial body moisture content and temperature respectively.

Conditions (14) and (15) allow equations (10) and (11) to be resolved on the assumption that the ceramic mix is a binary mixture of ceramic material and electrolyte. This assumption is conformed to by restricting the range to the period of decisive changes in the body volume, i.e. periods I and II of the drying process.

SOLUTION OF THE MATHEMATICAL MODEL FOR CONSTANT MATERIAL CHARACTERISTICS

On the assumption of constant material characteristics λ, ϱ, c_p, D_{ef}, equations (10) and (11) have the form

$$\partial_t C = D_{\mathrm{ef}}\, \partial_{xx} C, \tag{16}$$

and

$$\partial_t T = a\, \partial_{xx} T + RI^2/\varrho c_p V, \tag{17}$$

where a is thermal conductivity defined as $a = \lambda/\varrho c_p$. On introducing this assumption, the equations for conditions (14) and (15) can be resolved analytically by Laplace's transformation method [6]. The solutions yield relationships allowing the time development of moisture and temperature profiles to be calculated:

$$\tilde{C} = \int_0^{\tau} \gamma_1\, \mathrm{d}\tau^* + 2 \sum_{n=1}^{\infty} \cos(n\pi\xi) \int_0^{\tau} (-1)^n \gamma_1(\tau)^* \,.\, \exp\left(-n^2 \pi^2 (\tau - \tau^*)\right) \mathrm{d}\tau^* \tag{18}$$

and

$$\tilde{T} = \int_0^{\tau'} (\gamma_2 + \gamma_3)\, \mathrm{d}\tau'^* + 2 \sum_{n=1}^{\infty} \cos(n\pi\xi) \,.\, \int_0^{\tau'} (-1)^n \gamma_2(\tau'^*) \,.\, \exp\left(-n^2 \pi^2 (\tau - \tau^*)\right) \mathrm{d}\tau'^*, \tag{19}$$

where the following dimensionless quantities have been introduced:

$$\xi = x/L; \qquad \tilde{C} = (C - C_0)/C_0; \qquad \tilde{T} = (T - T_0)/T_0;$$
$$\tau' = at/L^2; \qquad \tau = D_{ef}t/L^2; \qquad \gamma_1 = \mathbf{h}_L\, L/D_{ef}\, C_0;$$
$$\gamma_2 = \mathbf{q}_L\, L/\lambda T_0; \qquad \gamma_3 = RI^2L^2/\lambda VT_0.$$

A comparison of the two equations shows that solution (18) is obtained from equation (18) by putting $\gamma_3 = 0$, i.e. by neglecting the source term.

SOLUTION OF THE MATHEMATICAL MODEL FOR MATERIAL CHARACTERISTICS DEPENDENT ON TEMPERATURE AND MOISTURE CONTENT

The previous case yielded a solution for constant material functions. However, this requirement in not met in practice and at least the dependence of the following material characteristics has to be taken into account:

$$D = D(T); \qquad a = a(C); \qquad \lambda = \lambda(C); \qquad c_p = c_p(C).$$

This situation makes the resolving of equations (10) and (11) for conditions (14) and (15) particularly complex. As an analytical solution of this problem is impossible, one of the numerical methods has to be employed.

The network method appears to be the most suitable one for resolving the problem [7]. The principle of the method is based on covering the region being studied with a time-space network with a division step of Δt and Δx. The nodes of the network are then given the respective moisture content and temperature values. Introduction of this network then allows differential relationships for substitution of differentials in equations (14) and (15) and in (10) and (11) to be obtained. On substituting by means of Crank—Nicolson's method, one obtains equations (10) and (11) in the form

$$\begin{aligned} -A_iC_{i-1}^{j+1} + (1 + A_i + A_{i+1})\, C_i^{j+1} - A_{i+1}C_{i+1}^{j+1} &= \\ = A_iC_{i-1}^{j} + (1 - A_i - A_{i+1})\, C_i^{j} + A_{i+1}C_{i+1}^{j}& \end{aligned} \tag{20}$$

$$\begin{aligned} -B_iT_{i-1}^{j+1} + (1 + B_i + B_{i+1})\, T_i^{j+1} - B_{i+1}T_{i+1}^{j+1} &= \\ B_iT_{i-1}^{j} + (1 - B_i - B_{i+1})\, T_i^{j} + B_{i+1}T_{i+1}^{j} + \mathrm{t}r^{j+1/2}/c_p\varrho,& \end{aligned} \tag{21}$$

where

$$\begin{aligned} A_i &= D_i^{j+1/2}\, \Delta t/\Delta x^2, \\ A_{i+1} &= D_{i+1}^{j+1/2}\Delta t/\Delta x^2, \\ B_i &= a_i^{j+1/2}\Delta t/\Delta x^2, \\ B_{i+1} &= a_{i+1}^{j+1/2}\Delta t/\Delta x^2. \end{aligned}$$

The bottom index designates the spatial dependence of the functions and the top one the time dependence of the functions. Supplementing with the boundary conditions

$i = 0$

$$T_0^{j+1} = \left\{T_1^{j+1} + \frac{2\mathbf{q}_L^{j+1/2}\Delta x}{\lambda_0^{j+1/2}} + T_1^j + T_0^j\left(\frac{\Delta x^2}{a_0\Delta t} - 1\right) + \right.$$

$$+ \frac{\Delta x^2 r^{j+1/2}}{\lambda_0}\Bigg\} \Bigg/ \left(1 + \frac{\Delta x^2}{a_0 \Delta t}\right), \tag{22}$$

$$C_0^{j+1} = \Bigg\{ C_1^{j+1} + \frac{2h_L^{j+1/2}\Delta x}{D_0^{j+1/2}} + $$
$$+ C_1^j + C_0^j\left(\frac{\Delta x^2}{D_0^{j+1/2}\Delta t} - 1\right)\Bigg\} \Bigg/ \left(1 + \frac{\Delta x^2}{D_0^{j+1/2}\Delta t}\right), \tag{23}$$

$i = N$

$$T_N^{j+1} = \Bigg\{ T_{N-1}^{j+1} + T_{N-1}^j + T_N^j + T_{N-1}^j\left(\frac{\Delta x^2}{a_N^{j+1/2}\Delta t} - 1\right) +$$
$$+ \frac{\Delta x^2 r^{j+1/2}}{\lambda_N^{j+1/2}}\Bigg\}\left(1 + \frac{\Delta x^2}{a_N^{j+1/2}\Delta t}\right), \tag{24}$$

$$C_N^{j+1} = \Bigg\{ C_{N-1}^{j+1} + C_{N-1}^j + C_N^j + C_{N-1}^j\left(\frac{\Delta x^2}{D_N^{j+1/2}\Delta t} - 1\right)\Bigg\} \Bigg/ \left(1 + \frac{\Delta x^2}{D_N^{j+1/2}\Delta t}\right), \tag{25}$$

yields, from equations (20—25), a set of linear equations with a tridiagonal matrix which can be resolved by some of the elimination methods, e. g. by the factorization method.

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ELEKTROODPOROVÉ SUŠENÍ KERAMIKY

Část I — Matematický model

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Je ukázán postup studia elektroodporového sušení keramického tělesa v plastickém stavu, založený na matematickém modelování. Řešení matematického modelu je provedeno pro konstantní materiálové veličiny metodou Laplaceovy transformace a řešení pro teplotně a vlhkostně závislé materiálové veličiny Crank—Nicolsonovou metodou sítí.

Obr. 1. Schéma průběhu EOS keramického tělesa.

ЭЛЕКТРОСОПРОТИВИТЕЛЬНАЯ СУШКА КЕРАМИКИ

I. Математическая модель

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В работе приводится способ исследования электросопротивительной сушки керамического тела в пластическом состоянии, основывающийся на математическом моделировании. Решение математической модели проводится в случае постоянной величины материала с помощью метода трансформации Лапласа и решение в случае величины материала, зависимой от температуры и влажности, с помощью метода сетей Кранк-Никольсона.

Рис. 1. Схема хода электросопротивительной сушки тела.

Recenze knih

O. L. ALTACH, P. D. SARKISOV: ŠLIFOVANIJE I POLIROVANIJE STEKLA I STEKLOIZDELIJ (Broušení a leštění skla a sklářských výrobků). Nakladatelství Vysšaja škola, Moskva 1988, 230 str., 115 obr., cena 40 kopejek (Kčs 5,—).

Recenzovaná kniha je učebnicí určenou pro střední odborná učiliště vychovávající kvalifikované pracovníky s odborností brusič skla, leštič sklářských výrobků a rytec skla. Tomuto účelu je přizpůsobena jak volba tematiky, tak úroveň jejího podání: autoři se omezují na popis zpracování užitkového skla (vázy, karafy, džbány, sklenice, popelníky, lustrové ověsky apod.) a téměř úplně vylučují fyzikálněchemické principy vysvětlovaných technologií včetně příslušného matematického aparátu.

První část knihy seznamuje čtenáře velice zhuštěnou formou s některými fyzikálními a chemickými vlastnostmi skla, s používanými surovinami, způsoby tavení a tvarování skla i jeho dalším zpracováním tepelným i mechanickým. Podrobněji jsou rozvedeny kapitoly týkající se abrazívních materiálů a nástrojů se zvláštním důrazem na diamantové nástroje. V závěru první části jsou probrány technologie opracování polotovarů bezprostředně předcházející konečnému zušlechtění výrobků: opukávání kopny, zapalování okrajů a jejich zabrušování, opracování dna, oddělování kopny plamenem.

Ve druhé části je soustředěno hlavní téma knihy — mechanické, chemické a tepelné zušlechťování užitkového skla. Poměrně podrobně jsou ve čtyřech kapitolách probrány jednotlivé technologie broušení a rytí (matové broušení, hladinářské broušení, kuličské broušení, rytí diamantovým hrotem atd.), chemického leštění, matování a leptání, potiskování a hlubokého leptání, leštění ohněm, ultrazvukového obrábění, nanášení oxidových a kovových vrstev, výroby barevných glazur.

Třetí část knihy probírá ve stručnosti základy normalizace užitkového skla, metody kontroly výroby, možnosti automatizace, organizaci výroby a bezpečnost práce.

Recenzovaná kniha je nepochybně cennou pomůckou pro studující sovětských odborných učilišť, kterým podává podrobné informace o postupech zušlechťování užitkového skla a navíc zjednodušený přehled o technologiích předcházejících zušlechťování. Pro čs. sklářskou veřejnost může být užitečná jako přehled příslušného sovětského strojního vybavení nebo odpovídající ruské terminologie. Čtenář hledající podrobnější popisy vlastních technologií však nepochybně sáhne spíše po domácích titulech — Broušení a leštění skla J. Götze nebo Mechanické opracování skla autorů Cozla a Streubelové z edice Hutní sklářská příručka.

J. Kavka

D. M. FREIK, M. A. GALUŠČAK, L. I. MEŽILOVSKAJA: FIZIKA I TECHNOLOGIA POLUPROVODNIKOVYCH PLJENOK (Fyzika a technologie polovodivých vrstev). Nakladatelství Višča škola, Lvov 1988, 150 str., 94 obr., cena 2,30 rublu (Kčs 29,—).

Poněkud obecně pojatý název knihy neprozrazuje čtenáři ihned její skutečné zaměření — příprava a vlastnosti epitaxních vrstev chalkogenidů olova a cínu. Toto zúžení tématu ovšem nikterak