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MODEL OF ELECTRIC RESISTANCE DRYING OF CERAMIC BODY IN QUASISTATIONARY CONDITIONS

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The proposed model of electric resistance drying of ceramic bodies in a quasistationary conditions was developed on the calculation of moisture and temperature fields. The solution has been derived or a constant diffusion coefficient as well as for a diffusion coefficient depending on temperature. A comparison of the results obtained with the two models and the experimental values showed a satisfactory agreement, thus proving the models to be suitable for the description of electric resistance drying.

INTRODUCTION

The purpose of drying of ceramic bodies is to remove the technologically necessary water from the ceramic mix as fast as possible without damaging the green ware. The uniformity of elimination of water from the body affects the final compactness and texture, and incorrect drying may be responsible for the formation of microcracks and even impaired integrity of the body. The uniformity of texture in the green body further influences the course of the consequent technological operation, that is firing, and likewise the final properties of the ware. The moisture field occurring in the body in the course of drying is therefore a suitable criterion for assessing the conditions of conducting the drying process. In order to control correctly the operation, one should thus know the moisture content field, as its character is decisive for the preparation of homogeneous, texturally unexceptionable bodies.

If a water-saturated ceramic mix is regarded as a binary mixture of incompressible components, i.e. water and the ceramic material, then electric resistance drying can be described as a combined process of moisture transfer by diffusion and heat transfer by conduction, while using an electric bulk source of heat.

On defining the volume fraction of moisture content C in the mix by the equation

$$C = w(\varrho_L \varrho_S^{-1} + w)^{-1}, \tag{1}$$

where ϱ_L is the density of water, ϱ_S is the density of the ceramic material and w is the absolute moisture content (i.e. the ration of water and solid component densities), one obtains the moisture balance in the form

$$\partial_t C + \operatorname{div} \boldsymbol{h} = 0, \tag{2}$$

where $\partial_t C$ is the derivative of moisture content in respect to time, and for the density of volume flow of moisture $\boldsymbol{h}$ it holds that

$$\boldsymbol{h} = -D_{ef} \operatorname{grad} C, \tag{3}$$

where D_{ef} is the effective diffusion coefficient including the effect of capillary barodiffusion while neglecting the effct of thermodiffusion on water transfer in the mix.

The thermal balance has the form

$$\varrho c_p \delta_t T = - \operatorname{div} \boldsymbol{q} + \varrho r, \tag{4}$$

where ϱ is the mix density, c_p is the specific heat, $\partial_t T$ signifies the derivative of temperature in respect to time and r is the specific heat source, while ϱr is the density of the bulk heat source. On neglecting the Dufour phenomenon, the following equation holds for the heat flux density $\boldsymbol{q}$:

$$\boldsymbol{q} = - \lambda \operatorname{grad} T, \tag{5}$$

where λ is the thermal conductivity of the mix.

If the electric heat source is of the internal type, the heat evolved is proportional to the electric curıent. As the water-saturated ceramic mix in an alternating electric fields behaves as an inhomogeneous conductor, it holds for the density of electric current j_e that

$$\boldsymbol{j}_e = -\sigma \operatorname{grad} \varphi - L_1 \operatorname{grad} c, \tag{6}$$

where σ is electric conductance, grad φ is the gradient of electric potential, c is the concentration of soluble salts and L_1 is the diffusion-electric coefficient. The diffusion electric phenomenon L_1 grad c in a water-saturated ceramic mix occurs as a result of a concentration gradient of water-soluble salts in the mix bulk.

The balance and constitutive equations (2) — (6) represent a general model of moisture and temperature transfer in a ceramic body with an internal heat source. Knowledge of the material quantities, supplemented with initial and boundary conditions defining the given operation, allows the time development of moisture content profiles and temperature profiles in the body to be calculated.

The process of electric resistance drying of ceramic bodies is divided into two stages. Following the stage of heating up the body, which creates a temperature and moisture content field, the drying proceeds at a constant rate, i.e. $\boldsymbol{h} = \boldsymbol{h}_s$. The internal heat source accelerates the heating up stage quite considerably as compared to convective heat transfer. From the standpoint of the course of electric resistance drying, the stage of constant drying rate is therefoıe of decisive significance. On the assumption that the temperature and moisture fields in the body have fully developed during the heating up stage, a precise solution of the stage of constant drying rate is provided by the so-called quasistationary state at which $\partial_t C = \partial_t \bar{C} =$ const., where $\bar{C}$ is the mean moisture content during time t. In the quasistationary state, the deflection of moisture content from its mean value is therefore independent of time. From this it follows that the shape of the moisture profile does not change in terms of time and that the local moisture content at each point decreases at the same rate, given by the rate at which the mean moisture content decreases.

The present study is concerned with developing and verifying a model of electric resistance drying of ceramic bodies under quasistationary conditions for the calculation of moisture and temperature profiles.

MODEL OF ELECTRIC RESISTANCE DRYING OF A PLATE-SHAPED BODY IN QUASISTATIONARY CONDITIONS

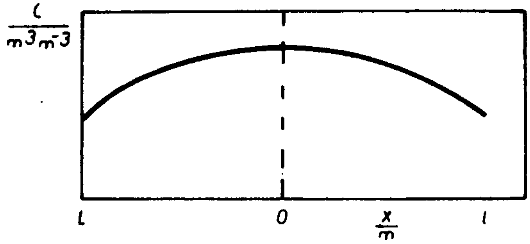
During electric resistance drying of an infinite plate $2L$ in thickness, the process involved is unidimensional diffusion and the balance equation (2) acquires the form

$$\partial_t C + \partial_x \boldsymbol{h} = 0, \tag{7}$$

where x is the coordinate in the direction of diffusion and for $\boldsymbol{h}$ it holds that

$$\boldsymbol{h} = -D\partial_x C. \tag{8}$$

If the conditions of drying are identical on both sides of the plate, the process is symmetrical (cf. Fig. 1) as the moisture profile is symmetrical with respect to the plate axis. Mathematically, this drying from both sides can be treated as one-side drying of a plate of half thickness [2, 6].



Fig, 1. Drying a plate 2L in thickness.

During the stage of constant drying rate, the flux of water at the plate surface is constant, so that it holds that

$$\boldsymbol{h} = \boldsymbol{h}_s \text{ for } x = L. \tag{9}$$

On defining the mean moisture content in the body at time t as

$$\bar{C}(t) = L^{-1} \int_0^L C(x, t)\, \mathrm{d}x, \tag{10}$$

then from equation (7) for the conditions (9) it follows that

$$d_t\bar{C} = L^{-1} \int_0^L \partial_t C\, \mathrm{d}x = -L^{-1} \int_0^L \partial_x \boldsymbol{h}\, \mathrm{d}x = -L^{-1}\boldsymbol{h}_s \tag{11}$$

as at $x = 0$, $\boldsymbol{h} = 0$. Integration of (11) yields

$$\bar{C} = \bar{C}_0 - \boldsymbol{h}_s L^{-1} t, \tag{12}$$

where $\bar{C}_0$ is the initial mean moisture content and $\boldsymbol{h}$ is the rate of drying defined by equation (9). For the sake of simplification, let us introduce the following dimensionless quatities:

$$\xi = x/L \qquad \tau = D_r L^{-2} t \qquad \delta = D/D_r,$$
$$\gamma = L^{-1}\boldsymbol{h}^{-1} D_r (C - \bar{C}), \tag{13}$$

where D_r is the standard diffusion coefficient, ξ, τ and δ are dimensionless distance, dimensionless time and dimensionless diffusivity respectively and γ is dimensionless moisture content for whose mean value with respect to (10) it holds that

$$\bar{\gamma} = \int_0^1 \gamma\, \mathrm{d}\xi = 0. \tag{14}$$

By introducing (8), (12) and (13) into (7) one obtains the equation

$$\partial_\tau \gamma = \partial_\xi (\delta(\gamma)\, \partial_\xi \gamma) + 1 \tag{15}$$

and the boundary condition after substitution into (8) and (13) has the form

$$\begin{aligned} \xi &= 0 \qquad \delta(\gamma)\, \partial_\xi \gamma = 0 \\ \xi &= 1 \qquad \delta(\gamma)\, \partial_\xi \gamma = -1. \end{aligned} \tag{16}$$

If the diffusion coefficient is independent of moisture content but generally a function of temperature or texture not subject to changes in terms of time, it is possible to consider a quasistationary state defined as follows:

$$\partial_t C = \partial_t \bar{C}, \text{ i.e. } \partial_\tau \gamma = 0. \tag{17}$$

In the quasistationary conditions, equation (7) acquires the following form with respect to (12):

$$\partial_x \boldsymbol{h} = \boldsymbol{h}_s L^{-1}, \tag{18}$$

whose integration

$$\boldsymbol{h} = \boldsymbol{h}_s L^{-1} x \tag{19}$$

shows that the flow of water by volume is a linear function of coordinate x. In view of (17), equation (15) acquires the form

$$\partial_\xi (\delta(\gamma)\, \partial_\xi \gamma) = -1. \tag{20}$$

The solution of (20) for conditions (14) and (16) has the form

$$\gamma = \int_0^1 \left[\int_0^\xi \delta(\xi)^{-1}\, \xi \,\mathrm{d}\xi\right] \mathrm{d}\xi - \int_0^\xi \delta(\xi)^{-1}\, \xi \,\mathrm{d}\xi. \tag{21}$$

This equation makes it possible to calculate the moisture profile in a body if the distribution of diffusivity in the plate $\delta(\xi)$, the mean moisture content in the body at time τ and the rate of drying $\boldsymbol{h}_s$ are known.

In a plate-shaped body L in thickness the heat transfer is unidimensional and balance (4) has then the form

$$\varrho c_p \partial_t T = -\partial_x \boldsymbol{q} + \varrho r, \tag{22}$$

where for $\boldsymbol{q}$ it holds that

$$\boldsymbol{q} = -\lambda \partial_x T. \tag{23}$$

During electric resistance drying, the body is heated with alternating electric current. If the ceramic mix is free of a concentration gradient of water-soluble salts, equation (6) for unidimensional electric current through the plate-shaped body acquires the form of Ohm's law:

$$\boldsymbol{j}_\mathrm{e} = -\sigma \operatorname{grad} \varphi, \tag{24}$$

and for the bulk heat source it holds that

$$\varrho r = R I^2 V^{-1}, \tag{25}$$

where R is the electric resistivity of the body having the volume V and I is the electric current. If ϱ, c_p and λ are constant, then for a steady state equation (22) has the form

$$\partial_x^2 T = -RI^2 V^{-1} \lambda^{-1}. \tag{26}$$

Introduction of dimensionless distance ξ according to (13) and resolving of (26) for the boundary conditions

$$\begin{aligned} \xi &= 0 \qquad \partial T/\partial \xi = 0, \\ \xi &= 1 \qquad\quad T = T_s, \end{aligned} \tag{27}$$

where T_s is the plate surface temperature,
yields the temperature profile in the body in the form

$$T = T_s + RI^2 L^2 (2V\lambda)^{-1} (1 - \xi^2). \tag{28}$$

The following equation holds for the mean body temperature:

$$\bar{T} = L^{-1} \int_0^L T(x)\,\mathrm{d}x = T_s + RI^2 L^2 (3V\lambda)^{-1}. \tag{29}$$

Equation (28) allows the temperature profile in the plate to be calculated if one knows the temperature of its surface, its electric resistivity, the passing electric current and its thermal conductivity.

SOLUTION OF THE MODEL FOR A CONSTANT DIFFUSION COEFFICIENT

The diffusion coefficient of water in a saturated porcelain mix does not depend on moisture content [3—4] and during the stage of constant drying rate, the body temperature is constant. On neglecting the temperature profile in the body one can introduce the assumption of a constant diffusion coefficient related to body surface temperature T_s. In the given case it holds that $D_r = D$ and $\delta = 1$. Equation (20) thus acquires the form

$$\partial_\xi^2 \gamma = -1, \tag{30}$$

and boundary conditions (16) have the form

$$\begin{aligned} \xi &= 0 \qquad \partial_\xi \gamma = 0 \\ \xi &= 1 \qquad \partial_\xi \gamma = -1, \end{aligned} \tag{31}$$

while condition (14) retains its form. Solution (30) for conditions (14) and (31) has the form

$$\gamma = 1/6 - \xi^2/2. \tag{32}$$

Substitution of (13) into (32) yields the following equation for the moisture profile in the body:

$$C(x) = \bar{C} + h_s L D^{-1} (1/6 - 1/2(x/L)^2). \tag{33}$$

If the condition $D = \text{const.}$ is met, the moisture and temperature profiles in the body during electric resistance drying in quasistationary conditions can be calculated from equations (33) and (28).

SOLUTION OF THE MODEL FOR THE TEMPERATURE-DEPENDENT DIFFUSION COEFFICIENT

In a quasistationary state, both the body temperature and the temperature profile are constant. The temperature dependence of the diffusion coefficient has the general form

$$D = D_0 \exp(-B/T), \tag{34}$$

where D_0 is a constant and B is the characteristic temperature of the process. If the temperature of the diffusion coefficient at body temperature T_s is chosen as the standard one, then it holds that

$$\delta = \exp(-B(T^{-1} - T_s^{-1})). \tag{35}$$

The parabolic temperature profile in the body is described by equation (28) which can be written in the form

$$T = T_0 + \xi^2(T_s - T_0), \tag{36}$$

where T_0 is the temperature at the plate centre.

The concentration profile is calculated by numerically resolving equation (21) while using equations (35) and (36).

VERIFICATION OF THE MODEL OF DRYING A CERAMIC PLATE IN QUASISTATIONARY CONDITIONS USING ELECTRIC RESISTANCE

The model and the validity of the simplifying assumptions introduced can be verified by comparing the calculated profiles $C = C(x)$ and $T = T(x)$ with the experimentally established ones. On the basis of the temperature dependence of the effective diffusion coefficient of water in porcelain mix in the form [5]

$$D_{ef} = (2.46 \,.\, 10^{-4} \exp(-2425/T))\ \mathrm{m^2\,s^{-1}}, \tag{37}$$

for the values of quantities $\bar{T} = 317.15$ K, $T_0 = 318.2$ K, $T_s = 315.85$ K, $\bar{C} =$ $= 0.4102\ \mathrm{m^3m^{-3}}$, $h = 2.6 \,.\, 10^{-7}\ \mathrm{ms^{-1}}$, $L = 15 \,.\, 10^{-3}$ m, $V = 5.4 \,.\, 10^{-5}\ \mathrm{m^3}$, $\lambda =$

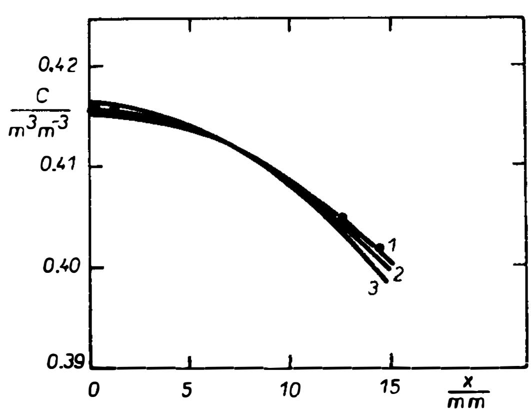


Fig. 2. Moisture content profile in a boedy;
1 — experimental, 2 — calculated according to (21), 3 — calculated according to (33).

$= 3.3\ \mathrm{Wm^{-1}\ K^{-1}}$, $U = 24.6$ V, $I = 0.18$ A, $R = 136.66\ \Omega$ the temperature profiles calculated from equations (33) and (21), and the temperature profiles calculated from equation (28) are plotted in Figs. 2 and 3. For the sake of comparison, the experimentally determined moisture and temperature profile in the body (while keeping the values of the quantities given above) are also plotted in Figs. 2 and 3.

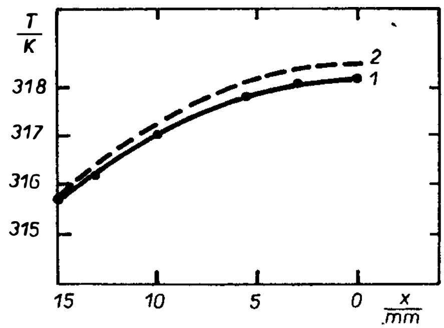


Fig. 3. Temperature profile in a body;
1 — experimental, 2 — calculated according to (28).

CONCLUSION

The following conclusions can be drawn from the experimental verification of the proposed model of drying a porcelain body in quasistationary conditions:

1. The satisfactory agreement of the experimental and calculated moisture and temperature profiles in the body showed the model to be suitable for describing the course of electric resistance drying of bodies in a quasistationary conditions.

2. In the case of small temperature gradients in the body, the model with a constant diffusion coefficient can be used for calculating the moisture profile with satisfactory accuracy.

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MODEL ELEKTROODPOROVÉHO SUŠENÍ KERAMICKÉHO TĚLESA V KVAZISTACIONÁRNÍM REŽIMU

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Elektroodporové sušení lze popsat jako kombinovaný proces sdílení vlhkosti difúzí a tepla vedením s vnitřním elektrickým zdrojem tepla. Je vypracován matematický model elektroodporového sušení keramického tělesa v kvazistacionárním režimu, umožňující výpočet časových

vývojů vlhkostních a teplotních polí v sušeném tělese. Výsledkem řešení pro případ konstantního difúzního koeficientu a difúzního koeficientu závislého na teplotě byl profil teploty a vlhkosti v tělese. Porovnání s profily stanovenými experimentálně je zcela uspokojivé a je tedy důkazem, že nalezený model vystihuje proces elektroodporového sušení.

Obr. 1. Sušení desky tloušťky 2L.

Obr. 2. Vlhkostní profil v tělese; 1 — experimentální, 2 — vypočtený podle (21), 3 — vypočtený podle (33).

Obr. 3. Teplotní profil v tělese; 1 — experimentální, 2 — vypočtený podle (28).

МОДЕЛЬ ЭЛЕКТРОСОПРОТИВИТЕЛЬНОЙ СУШКИ КЕРАМИЧЕСКОГО ТЕЛА В КВАЗИСТАЦИОНАРНОМ РЕЖИМЕ

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Электросопротивительную сушку можно описать как комбинированный процесс передачи влажности диффузией и тепла передачей с внутренним электрическим источником тепла. Была разработана математическая модель электросопротивительной сушки керемического тела в квазистационарном режиме, предоставляющем возможность рассчета временных развитий влажностных и температурных полей в сушенном теле. Результатом решения в случае постоянного коэффициента диффузии и коэффициента диффузии, зависящего от температуры, является профиль температуры и влажности в теле. Сопоставление с профилями, установленными экспериментальным путем, является весьма надежным, а следовательно служит и доказательством, что найденная модель отражает процесс электросопротивительной сушки.

Рис. 1. Сушка пластинки толщиной в 2L.

Рис. 2. Влажностный профиль в теле; 1 — экспериментальный, 2 — рассчитанный согласно (21), 3 — рассчитанный согласно (33).

Рис. 3. Температурный профиль в теле; 1 — экспериментальный, 2 — рассчитанный согласно (28).