CERAMICS — Silikáty 34, s. 343—347 (1990)

APPLICATION OF STATISTICAL IDENTIFICATION TO PHYSICAL MODELS OF GLASS FURNACES; I

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Received 9. 1. 1990

The suggested method is based on investigating the dynamic properties of a glass furnace and had the purpose to obtain basic data for adaptive control of furnaces. The procedure simplifies the relationships between the technological quantities and allows random signals to be utilized for changing the input quantities such the electric power input. Determination of the input quantities and their evaluation is an important part of the statistical identification method. The standard least square method and the extended least square method were used to evaluate the input signals, emploing calculation by the elementary rotation method and the root filter method. The results of statistical identification were verified by means of criteria such as the prediction test, the normal noise test and the white noise test.

THE PROBLEMS OF STATISTICAL IDENTIFICATION

In the authors' opinion, experimental identification is more suitable than mathematico-physical analysis for investigating the dynamic behaviour of glass tank furnaces. Experimental identification can be defined as a process during which a mathematical model is created on the basis of measurements, while also determining the parameters and shape of the model. This is selected from a certain pre-determined class of models on the basis of input and output data so as to ensure its equivalency with the process being studied. In selecting the model one should also take into account the principle of the process being identified and the purpose the model is to serve.

The present study deals with stochastic linear one-dimensional discrete model with concentrated parameters in the form of external description, because only discrete values and not continuous records of input and output data can be obtained from a physical model. This is why a discrete system has been chosen for describing the behaviour of a physical model of glass furnace, using a model in the form of differential equation (1)

$$y(k) + \sum_{i=0}^{n} a_i y(k-i) = \sum_{i=0}^{n} b_i u(k-i) + e_r(k), \tag{1}$$

where $e_r(k)$ is the error of the equation.

Using polynomial operators and by introducing a symbol for shift by i intervals to the right, equation (1) can be written in the form

$$[1 + A(z^{-1})]\, y(k) = B(z^{-1})\, u(k) + e_r(k), \tag{2}$$

where $u(k)$ is the input signal and $y(k)$ is the output signal in k-th moment, and n is the order of the system.

Statistical identification has the purpose to establish, on the basis of measuring $u(k)$ and $y(k)$, an estimate of the coefficients of polynomials $A(z^{-1})$ and $B(z^{-1})$ so as to get the coefficients as close as possible to the actual parameters of the system studied. The principle of deriving the coefficients of polynomials $A(z^{-1})$ and $B(z^{-1})$ indicates that use of the Z-transformation would be disadvantageous (cf equation (2)) as a shortening of the sampling period would yield less information on the system, in spite of the fact that more frequent sampling increases the information capacity in the sample sequence. On the other hand, lengthening of the sampling period results in gradual loss of information on components with small time constants as well as on oscillating high-frequency components. A suitable sampling period can be determined from the sampling theorem

$$f_{\mathrm{m}} = 1/2\,T, \tag{3}$$

where T is the sampling period, and

f_{m} is the maximum frequency contained in the signal.

The study made use of a discrete description of the stochastic system where the error of the equation was considered to result from filtering the white noise by the ARMA system (Auto Regress Moving Average) as follows from equation (4):

$$[1 - C(z^{-1})]\, e_r(k) = D(z^{-1})\, n^*(k). \tag{4}$$

As regards the stochastic signals employed, only their statistical characteristics were known, namely the mean value, dispersion (scatter), stationarity, correlation function and output spectral density. In statistical identification, a decisive role is played by spectral properties of the input signal which should correspond to the frequency characteristics of the system in question. Considerable requirements are put forward for the uniformity of the frequency spectrum because each real system acts as a high-frequency filter and about the same applies to the low-frequency region. These demands are best met by the PRBS signal (pseudo-random binary signal).

The parameters of the differential equation were estimated by the least square (LS) method and the extended least square method (ELS), and resolved by the elementary rotation method and the root filter method. For N measurements, one obtains a pre-determined system of equations which can be expressed in matrix form,

$$Y = Za - \varepsilon_r. \tag{5}$$

Owing to the error vector of the system of equations (ε_r), the estimate of $\hat{a}$, i.e. the vector of the estimated parameters of polynomials $A(z^{-1})$ and $B(z^{-1})$ of the system, is by itself a random quantity, so that it must be demanded that

$$E(\hat{a}) = a, \tag{6}$$

i.e. that the mean estimate value be precisely equal to the value of the parameters being estimated. Only then is the estimate unbiased. The output signal and the equation error correlate, so that deviated estimates of the polynomial parameters are obtained; they deviate from the actual values proportionally to the decreasing signal/noise ratio of the output signal $y(k)$. The ELS method is based on the principle of estimating the ARMA noise model (cf. equation (4)) where the coefficients are estimated directly by adjusting the basic algorithm of the LS method.

SIGNAL TESTING DURING STATISTICAL IDENTIFICATION

1. Prediction test

The term prediction is understood to mean calculation of the system output signal by means of calculated values and estimated coefficients while taking into account the individual component (trend). Then

$$y(k) = -\sum_{i=0}^{n} a_i y(k-i) + \sum_{i=0}^{n} b_i y(k-i) + \sum_{i=0}^{m} c_i k_i, \tag{7}$$

where m is the polynomial degree expressing the trend.

On the basis of equation (1) containing the quantity $e_r(k)$, the latter can be interpreted as the prediction error, i.e. the difference between the actual value of the output and its estimate (prediction):

$$e_r(k) = \hat{y}(k) - y(k). \tag{8}$$

This error of the prediction can then be expressed statistically in various ways, e.g. as a mean relative quadratic error (R1) or as a relative linear prediction error (R3):

$$R1 = \sum_{k=m_3}^{n} \frac{e_r(k)^2}{y(k)^2}, \tag{9}$$

$$R3 = \sum_{k=m_3}^{n} \frac{e_r(k)}{y(k)}, \tag{10}$$

where m_3 is the value from which the algorithm has been started and n is the number of samples (data).

2. Normal noise test

This is understood to mean Gaussian noise distribution. It can be applied to the error of equation (1) or to the prediction error (8). Its value should be as low as possible:

$$\lambda = \sqrt{\frac{1}{n-L} \sum_{i=1}^{n} e_r(i)}, \tag{11}$$

where L is the overall number of parameters of the model selected and n is the number of samples (data).

3. White noise content test

Its value for a given noise order should be as low as possible. It is calculated according to the equation

$$w_e = \frac{1}{\sum_{i=0}^{n} \frac{\varphi_{ee}i}{\varphi_{ee}0}}$$

where φ_{ee} is the autocorrelation function for source noise.

CONCLUSION

The standard least square method and that of extended least squares are used in statistical identification to determine the coefficients of the differential equation describing the dynamic behaviour of the system. The predetermined system of equations can be solved by the elementary rotation method or the root filter method. Application of these methods to mathematical models of glass furnaces allows information on their dynamic behaviour to be obtained relatively rapidly and at low cost without affecting the operational stability of the furnaces. An additional advantage is provided by the fact that in the processes mentioned above, use can be made of pseudo-random binary signals which are uniform from the standpoint of the frequency spectrum. Utilization of physical models of glass furnaces in the investigation of their dynamic behaviour is also convenient because a physical model provides very precise relations between the significant technological quantities such as power input, temperature in the furnace, withdrawal of melt from the furnace, flow in the furnace, etc.

References

[1] Eykhoff R. W.: *The Principles of Identification* (in Russian), Mir, Moscow 1975.
[2] Fabian V.: *Elementary Statistical Mathematics* (in Czech), SNTL, Prague 1971.
[3] Drábek O., Macháček J.: *Experimental Identification and Control of Systems* (in Czech, textbook), VŠCHT, Pardubice 1983.
[4] Peterka V.: Automatica *17*, 41 (1981).
[5] Strejc V.: Automatica *17*, 7 (1981).
[6] Ashfield A. W.: Furnace Control Strategies, in: *Proc. XIIth Int. Congress on Glass*, p. 943. Albuquerque 1980.

NÁVRH STATISTICKÉ IDENTIFIKACE FYZIKÁLNÍHO MODELU SKLÁŘSKÉ TAVICÍ PECE

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K vyšetřování dynamického chování sklářské tavicí pece na jejím fyzikálním modelu bylo použito experimentální identifikace, pomocí níž se na základě měření vytváří matematický model, přičemž dochází jak k určování parametrů modelu, tak i jeho tvaru. Pro popis chování fyzikálního modelu sklářské tavicí pece byla volena diskrétní soustava s užitím modelu ve tvaru diferenční rovnice (viz (1) a (2)). K určení odhadu koeficientů v rovnicích (1) a (2) byla použita metoda statistické identifikace obsahující metodu nejmenších čtverců a rozšířenou metodu nejmenších čtverců.

Stejně jako na peci nelze na fyzikálním modelu získat spojité záznamy vstupních a výstupních veličin, ale pouze diskrétní hodnoty, a proto byl v práci použit diskrétní popis soustavy jako stochastické. U stochastických signálů jsou známy pouze jejich stochastické charakteristiky. Zde byly užity střední hodnota, rozptyl, korelační funkce a výkonová spektrální hustota. Zvláště spektrální vlastnosti signálů jsou důležité a musí odpovídat frekvenční charakteristice proměřované soustavy. Těmto nárokům nejlépe odpovídá pseudo-náhodný binární signál.

Kvalita použitých signálů při statistické identifikaci byla testována pomocí tří testů. Byly to:
— test predikce, kde chyba predikce byla statisticky vyhodnocena jako průměrná kvadratická chyba predikce (rovnice (9)) a jako poměrná lineární chyba predikce (rovnice (10)),
— test normálního šumu, zahrnující rozdělení šumu podle Gausse (rovnice (11)),
— test na obsah bílého šumu (rovnice (12)).

Navržená metodika vyšetřování dynamického chování sklářské tavicí pece na fyzikálním modelu má proti měření na peci několik výhod, neboť:
a) fyzikální model poskytuje velmi přesné vztahy mezi významnými technologickými veličinami, např. příkon do pece, teplota v peci apod., a jeho provoz je laciný;
b) fyzikální model umožňuje proměřit dynamické vlastnosti tavicí pece, které nelze, z technických důvodů, uskutečnit v provozních podmínkách;
c) provoz fyzikálního modelu je možno odklonit od ustáleného technologického režimu bez problémů, což by na peci mohlo vést k řadě těžkostí.

ПРОЕКТ СТАТИСТИЧЕСКОЙ ИДЕНТИФИКАЦИИ ФИЗИЧЕСКОЙ МОДЕЛИ СТЕКЛОВАРЕННОЙ ПЕЧИ

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Для исследования динамического поведения стекловаренной печи на ее физической модели использовали экспериментальную идентификацию, с помощью которой на основании измерений создается математическая модель, причем проводится как определение параметров модели, так и ее формы. Для описания поведения физической модели стекловаренной печи подбирали дискретную систему с использованием модели в виде дифференциального уравнения (см. (1) и (2)). Для определения коэффициентов в уравнениях (1) и (2) использовали метод статистической идентификации, основывающийся на методе наименьших квадратиков и на распространенном методе наименьших квадратиков.

Подобно как на печи так и на физическом модели нельзя получить непрерывные записи входных и выходных величин, а только дискретные величины, и поэтому нами в работе используется дискретное описание системы как системы стохастической. У стохастических сигналов известны только их стохастические характеристики. В нашем случае были использованы средняя величина, рассеяние, функция корреляции и мощностная спектральная плотность. Особенно важны спектральные свойства сигналов и они должны соответствовать частотной характеристике измеряемой системы. Этим требованиям лучше всего соответствует псевдо-бинарный сигнал.

Качество используемых сигналов при статистической идентификации проверяли с помощью трех тестов, а именно:
— с помощью теста предикции, где погрешность предикции статистически рассматривается как средняя квадратическая погрешность (уравнение (9)) и как линейяая погрешность предикции (уравнение (10)),
— с помощью теста нормального шума, заключающего разделение шума согласно Гауссу (уравнение (11)),
— с помощью теста на содержание белого шума (уравнение (12)).

Предлагаемая методика рассмотрения динамического поведения стекловаренной печи на физической модели обладает по сравнению с измерением на печи несколькими преимуществами, так как:
а) физическая модель предоставляет весьма точные отношения между важными технологическими величинами, напр. подводимая мощность в речь, температура в печи и т. д. и ее производство дешево;
б) физическая модель предоставляет возможность измерения динамических свойств стекловаренной печи, которые нельзя, по техническим причинами, проводить при производственных условиях,;
в) эксплуатацию физической модели можно отклонить от установленного технологичесъого режима без проблем. что на печи могло бы вызвать ряд затруднений.