CERAMICS — Silikáty, 34, s. 9—19 (1990)

APPLICATION OF STATISTICAL IDENTIFICATION TO PHYSICAL MODELS OF GLASS FURNACES; II

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Received 9. 1. 1990

From the mathematical point of view and with respect to control, a glass furnace can be regarded as a multidimensional system which can be described by appropriate differential equations. Estimation of the parameters of the differential equations was verified on a physical model of a glass furnace and the dynamic properties of the furnace were measured on the model by selected jump changes of the input electric power. The responses of the output variables, i.e. temperatures at the chosen points of the physical model of the melting furnace section, were studied by application of a pseudo-random binary signal to the input. The estimates of the differential equation parameters were likewise established by calculation, using the least square method and the extended least square method. The final results describe very satisfactorily the dynamic behaviour of the furnace.

INTRODUCTION

A glass melting furnace is a multidimensional system with significant input and output quantities such as power input, rate of batch feeding, glass melt temperature, rate of melt withdrawal, etc. A model including all of the important input and output quantities would be complex and very difficult to resolve. It is therefore usually simplified to a single-dimensional system called SISO (single-input single-output), where electric input is the sole input quantity and all the others influencing the output quantities are regarded as defect quantities. Glass melt temperature (that of the model liquid) is the most frequently employed output quantity which, however, depends considerably on the point in the furnace chosen for its measurement. The values of current and voltage between the electrodes and the respective resistance, corresponding to a certain temperature, may also be used as an output quantity.

Identification of the system model, i.e. determination or calculation of the coefficients in equation (1),

$$[1 + A(z^{-1})]\, y(k) = B(z^{-1})\, u(k) + e_r(k) \tag{1}$$

by means of some of the statistical identification methods, must precede the proposal of the control mechanism. To yield satisfactory results, the statistical identification has to make use of such input signals which ensure adequate excitation of the system in the frequency range affecting significantly the dynamic behaviour of the system. Choice of a suitable period for sampling the system is also important. Application of pseudo-random binary signals at the system input during changes in the electric power input appears to be the most convenient method. However, use of such a signal on an actual furnace would mean a rough interference with the technological schedule of the furnace. This disadvantage can be eliminated in two ways: the calculations can either be carried out from operational technological records, or the measurements are performed on a physical model of the furnace which simulates the behaviour of the actual furnace.

DESCRIPTION OF THE PHYSICAL MODEL

The methods of statistical identification involving the testing of input and output signals and verification of the final results were employed on a physical model of an all-electric tank furnace of type Gell, made on a 1 : 15 scale. The model liquid simulating by its properties the SIMAX glass melt was heated by passage of electric current, introduced into the liquid through plate- and rod-shaped electrodes placed in the side walls of the model. The model liquid at the required temperature was fed into the model by a plunger pump. The feeding circuit also included a flowmeter, ensuring supply of a constant amount of the liquid into the model in the course of the experiment. Copper-constantan thermocouples placed in the model (Fig. 1) and connected to an automatic change-over switch provided temperature measurement. The temperatures were recorded continuously by a printer.

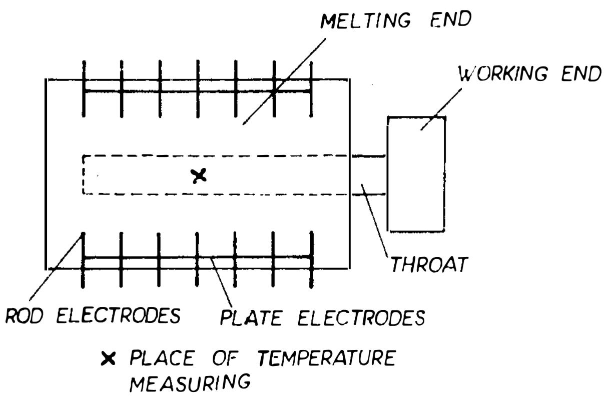

Fig. 1. Position of the measuring thermocouple in the glass furnace model.

MEASURING THE TRANSITION CHARACTERISTIC OF THE SYSTEM

The first important information on the dynamic behaviour of the system is provided by measuring the transition characteristic as a response to an input jump. The transition characteristic thus obtained then yields another significant characteristic for the design of the experiment, namely the time constant of the system T_s, sometimes also designated $\tau_{\max}$. The time constant established allows the input signal to be estimated, in the case of a pseudo-random binary signal jointly with its change period and its sampling period T. In the course of experiments with measuring the transition characteristic, an input jump from 60 W to 70 W was used. The time constant of the model was established from three approximation measurements and its values amounted to 120 minutes (Fig. 2). The measurement was always concluded as soon as the system attained a steady state.

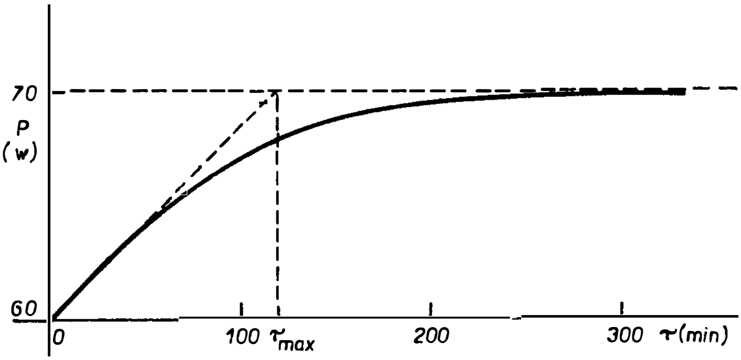

Fig. 2. Transition characteristics of the system.

DESIGN OF THE INPUT SIGNAL

The sequence length P_t and the interval of changes in the pseudo-random binary signal Δt are important factors for the actual selection of the pseudo-random binary signal. The following equations (2) and (3) hold for the two respective factors:

$$\Delta t < 0.314\ \tau_{\max}, \tag{2}$$

$$P_t > 12.56\ \tau_{\max}/\Delta t. \tag{3}$$

For the time constant $\tau_{\max}$ established for the furnace model employed, it is convenient that $P_t > 40$ and $\Delta t < 37.7$ minutes. From the value of $\tau_{\max}$ it is also possible to calculate the maximum frequencies arising in the system. For a signal to cover both frequencies, it must hold that

$$f_d < f_{\min} \quad \text{and} \quad f_h > f_{\max}, \tag{4}$$

where f_d is the bottom frequency of the input signal and f_h is the top frequency of the input signal. With the use of the pseudo-random binary signal, the para-

Table I
Parameters of input signals

Set No.	P_t	t [min]	f_d^{-1} [min]	f_h [min]	T [min]	El. input [W]	PRBS value	
1	92	20	5.1×10^{-4}	0.025	16	80—75	1	0
2	56	15	1.2×10^{-4}	0.033	12	80—75	1	0
3	46	10	2.1×10^{-4}	0.050	8	80—75	1	0
4	38	10	2.6×10^{-4}	0.050	8	80—75	1	0
5	35	15	1.9×10^{-4}	0.033	12	80—75	1	0
6	30	15	2.2×10^{-4}	0.033	12	82—75	1	0

meters of the input signals employed in the measurements on the physical model are listed in Table I together with the sampling period T chosen.

The physical model was always in the steady state before starting the measurement of data for identification. The electric input for the model electrodes was chosen as the input signal and temperature at a chosen thermocouple as the output signal. A pseudo-random binary signal generating the values of 0 and 1 was applied to the model input so that the value 0 corresponded to the input of 75 W and the value 1 to electric input of 80 W or 82 W. The change of power input by 5—10 % in steady state was chosen so as to bring about an adequate response at the system output but at the same time to avoid transgression of the system to its non-linear region in view of the linear model employed.

TESTING THE SIGNALS BY STATISTICAL METHODS

All of the experimental data sets were tested by autocorrelation and correlation functions (R_{xx}, R_{xy}, R_{yz}) as well as by output spectral density S_{xx}. The mean value of the signal was also determined.

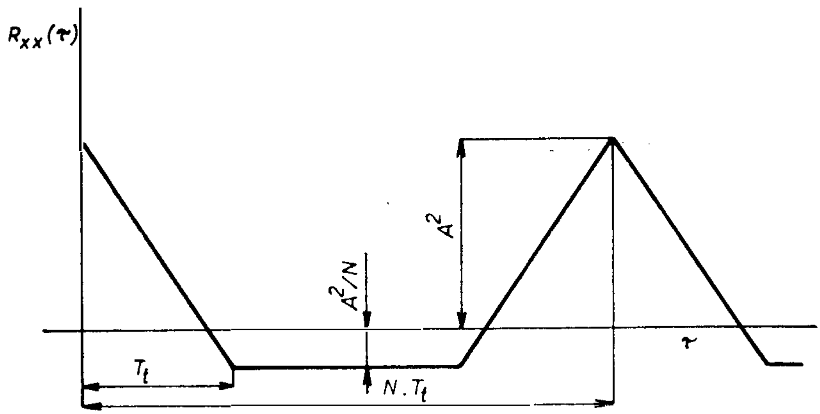

Fig. 3. Autocorrelation function of the pseudo-random binary signal.

The autocorrelogram of the pseudo-random signal (Fig. 3) corresponded by its course to a theoretical random phenomenon and amplitude A^2 had the value of 0.2499. The low value of output spectral density ($S_{xx} < 0$) proves that the sequence does not contain any unidirectional component. The autocorrelograms of the input signals of sets 1, 2 and 3 are not shown as their course was similar to that of the pseudo-random binary signal and exhibited increasing dependence of the output signal on the input one. The shape of the course of the output spectral density for set 1 was similar to that resulting from the pseudo-random binary signal (Fig. 4) and its value at point $f = 0$ did not indicate the content of the unidirectional component. The shape of the output spectral density for sets 2 and 3 also resembles the course of S_{xx} for the pseudo-random binary signal and is constant in the frequency band $f_d = 0.025\ \mathrm{min}^{-1}$ and $f_h = 0.0375\ \mathrm{min}^{-1}$. The values of S_{xx} (0) and the rapid decrease of the subsequent values indicated to a high content of the unidirectional component. In the case of set 4, the estimate of the statistical properties of the signal was proved unsatisfactory as it length was too small.

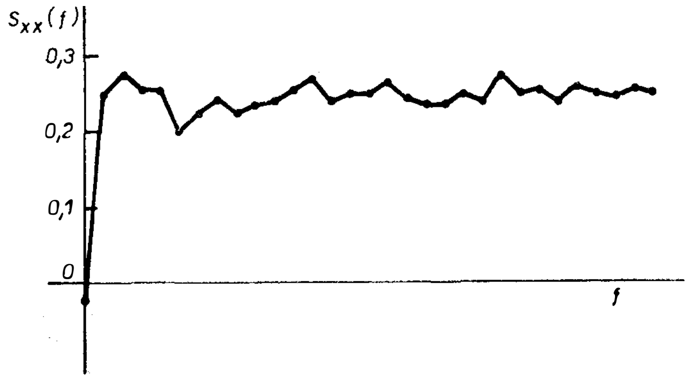

Fig. 4. The course of output spectral density (model No. 1).

THE STAGES AND PROCEDURE OF IDENTIFICATION

The input sets were proposed so as to ensure their meeting the respective requirements and to allow for their testing by statistical methods after the measurement. The input and output signals were processed by the statistical identification method with a predetermined order of the mathematical model, the trend coefficient and the calculation method. All of the results obtained were verified. From this it follows that the best describing models can be used either separately or as averaged models based on these results.

VERIFYING THE MODELS OBTAINED FROM STATISTICAL IDENTIFICATION

The term verifying a mathematical model is understood to mean the testing of results obtained from statistical identification in a way ensuring summarization and providing evidence on the fact that the model actually describes the behaviour of the system and corresponds to it by its order. The main criteria employed were the prediction error expressed as the mean prediction error R, the mean relative quadratic prediction error R1 and the mean linear prediction error $R3$, whose values should ideally be zero. It is also possible to interpret directly the predicted value of output signal $y(k)$ at the k-th moment, which should be identical with the value of the output signal measured at the k-th moment. For a decision on the suitability of a chosen order of the model it is possible to use the test for the content of white noise and that for normal noise distribution: the values obtained should also be as low as possible.

In evaluating the models describing dynamic behaviour of a system it is important to evaluate the criteria in a complex way. On this basis, altogether six models were selected, and their coefficients of the differential equation are summarized in Table II.

Table II

Values of the coefficients of the differential equation

Model No.	A_1	A_2	A_3	B_0	B_1	B_2
1	0.00755	—0.0111	0.0163	0.0173	—0.00464	0.173
2	—0.0258	0.0136	0.0119	0.025	—0.0281	0.00572
3	0.0244	—0.0135	0.0206	0.0097	—0.0181	0.0106
4	—0.00393	0.014	0.0244	0.0154	0.00602	—0.0052
5	0.00517	0.0154		0.0281	—0.0137	
6	—0.00421	0.0316		0.029	—0.0284	

The resultant models were converted (by means of the ZPT program) to an external continuous description of the system where the individual coefficients of the polynomial $A(z^{-1})$ and $B(z^{-1})$ were the input variables. For the models listed in Table 2, the results are summarized in Table III. The $A(z^{-1})$ polynomials were tested for stability in the complex plane by the STABPOL program and found to be situated in the stable region, because the absolute value of the criterion was always lower than unity. The results are summarized in Table IV.

Table III

Coefficients of the continuous description of the system

Model No.	1	2	3	1	2	3
1	0.05427	—0.07514	0.08784	0.0947	0.08624	0.06809
2	0.0441	0.3536	1.166	0.02717	0.03457	0.05036
3	0.05595	0.4393	1.468	0.09046	0.08023	0.06669
4	0.04875	0.3793	1.379	0.02417	0.02333	0.0128
5	0.1291	0.6588		0.00409	—0.00131	
6	0.1169	—0.0564		0.00637	0.00753	

Table IV

Values of the stability criterion

Model No.	Stability criterion
1	7.82×10^{-3}
2	2.56×10^{-2}
3	2.45×10^{-2}
4	4.23×10^{-3}
5	5.09×10^{-3}
6	4.08×10^{-3}

DISCUSSION OF THE RESULTS

A physical model of an all-electric glass tank furnace was employed for measuring the dynamic properties of the melting furnace which at present are very difficult to determine under operational conditions and with the available technical means, because any deviation from the standard technological schedule would be extremely uneconomical and could lead to a number of additional difficulties. For these reasons, the method of statistical identification was applied to a physical model of the Gell furnace, intended for melting the SIMAX glass. The model simplifies the relationships between the technological quantities and allows the pseudo-random binary signal to be employed for selected ranges of abrupt changes in input, while respecting the conditions and requirements for the frequency spectrum of the signal. Temperatures at several preselected points in the furnace were considered as output quantities. The data measured by a thermocouple at the centre of the melting tank zone, i.e. the region of maximum temperature, was chosen for the calculation proper of the mathematical model (determination of the coefficients of the differential equation), describing the dynamic behaviour of the furnace.

The methods of statistical identification (LS and ELS) were employed in the determination of the coefficients of the differential equations describing the dynamic behaviour of the system. Better results were obtained from the ELS method. The chosen order of the differential equation, equal to three, was proved correct by verification of the resultant models according to the prediction error and the test for the content of white noise w_e, whose value amounted to 0.65, and by the test for normal noise distribution λ.

The overall evaluation of criteria employed in verification of the mathematical models of the system in question in the form of a differential equation (Table II) indicates that the experimental data sets yielded results describing very well the dynamic behaviour of the system. The models were then transformed into a continuous region by means of the ZPT program and are given in Table III. As negative results were obtained for some models by transformation into a continuous form (the models described inadequately the dynamic behaviour of the system and are not shown), the stability of the polynomial in the complex plane was tested by the STABPOL program with all of the models. The testing showed that for models situated in the unstable region of the complex plane (where the absolute value of the stability criterion is larger than, or equal to, unity), it is impossible to use the given way of transforming the model to the continuous region by the ZPT program. In the case of the transformed models (Table II) the polynomials were in the stable region of the complex plane, because the absolute value of the stability criterion was always lower than unity. The results are summarized in Table IV.

The effect of errors of the temperature measuring devices (1 °C) was found to be the main source of errors, as one should keep in mind that the data processing was effected by mathematical apparatus which is highly accurate. The computer technology employed likewise did not provide any major sources of errors. A comparison of the verification results (Table II), where the relative linear prediction error amounted to 1.5 % (0.65 °C) and the measuring errors to 1 °C, allows to conclude that the measuring equipment can affect the results of statistical identification to a substantial degree.

The estimation of coefficients for the differential equation of the system is positively affected by the length of the experimental data sequence which permits the properties of the statistical methods to manifest themselves fully. This can be observed on the course of the prediction errors (Figs. 5 and 6) which should approach zero as a limit. Positive use can also be made of the coefficient of exponential forgetting in the mathematical apparatus, which makes the model adapt itself more rapidly to changes in technological conditions. The coefficient of exponential forgetting should of course not exceed a minimum value with respect to the sequence length.

The models established hold over a range where linear behaviour can be expected with respect to the requirements for the model formulated in its selection, namely for small changes in the electric power input of up to about 5 to 10 %.

Because of the dispersion of the experimental output parameters, electric resistivity between the electrodes was found to be more convenient, as it could be measured with higher accuracy than temperature by means of thermocouples. The second reason why the resistivity between electrodes was preferred as a value

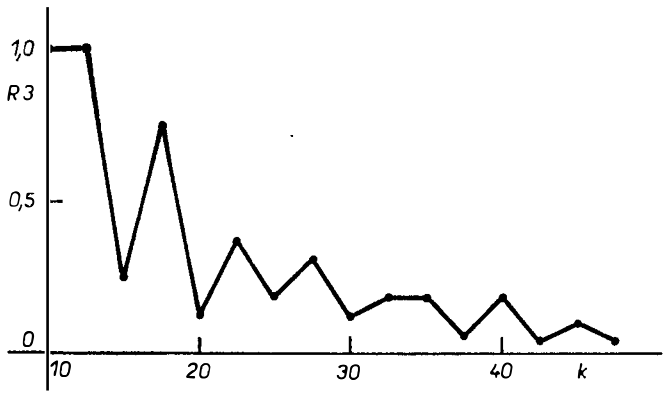

Fig. 5. The course of the mean relative linear prediction error (model No. 3).

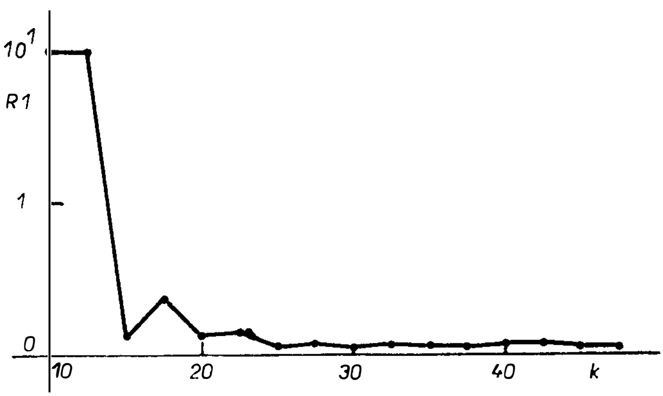

Fig. 6. The course of the mean relative quadratic error of prediction (model No. 3).

determining the glass melt temperature was the fact that temperature is only rarely measured directly in the glass melt in actual tank furnaces, where it is mostly assessed on the basis of temperatures measured in the crown, the tank side walls, etc.

Measurements on the physical model also demonstrated that both the melting furnace and its physical model showed a tendency to non-linearity. For this reason, it is recommended to attach a computer to the furnace as well as to the model in order to calculate continuously the constants of the mathematical model of the system. In this way one obtains a highly advantageous adaptive control system utilizing in-line identification, based on the records of time sequences of technological data.

CONCLUSION

A physical model of an all-electric tank furnace was used to determine its dynamic behaviour, and the respective mathematical models in both discrete and continuous form were devised. The constants of the suggested mathematical models in the form of differential equations were calculated by the methods of statistical identification (LS and ELS). The models were verified on the basis of selected criteria. The models obtained were transformed into a continuous description and tested for stability in the complex plane

The mathematical models calculated describe very well the dynamic behaviour of the system and can be applied to the actual furnace. It has been proved advantageous to connect a computer to the physical model or the furnace, and to calculate continuously the constants of the model, thus improving their accuracy according to the changing conditions.

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APLIKACE STATISTICKÉ IDENTIFIKACE NA FYZIKÁLNÍ MODEL SKLÁŘSKÉ TAVICÍ PECE

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Na fyzikální model celoelektrické sklářské tavicí pece byla aplikována metoda statistické identifikace poskytující informace o dynamickém chování pece pro potřeby jejího řízení. Dynamické vlastnosti pece byly vyšetřovány na základě odezvy výstupní veličiny (teplota) na skokové změny veličiny vstupní (elektrický příkon). Po vyšetření vlastností vstupních signálů byl na vstupu používán pseudo-náhodný binární signál. Metodami statistické identifikace (metoda nejmenších čtverců a rozšířená metoda nejmenších čtverců) byly určeny jak koeficienty diferenčních rovnic, tak i jejich řád, který se rovná 3. Tyto hodnoty byly potvrzeny verifikací výsledných modelů podle testů.

Po vyhodnocení matematických modelů soustavy na základě hodnotících kritérií bylo možné konstatovat, že dosažené výsledky dobře vystihují dynamické chování soustavy. Při převodu některých modelů z diskrétní do spojité oblasti bylo však dosaženo negativních výsledků, a proto byla u všech modelů testována jejich stabilita programem STABPOL.

Během měření byly také odhaleny zdroje chyb, které mohou negativně ovlivnit výsledky statistické identifikace. Jako nejdůležitější zdroj chyb byly určeny chyby měřicího zařízení (např. teploty). Tento nedostatek byl odstraněn postupem, navrženým autory, na jehož základě bylo doporučeno používat k určení teploty elektrický odpor mezi elektrodami, jehož měření je mnohem přesnější. Výhodným se rovněž ukázalo připojení počítače k modelu, který umožňuje neustále dopočítávat konstanty modelu a zpřesňovat je podle měnících se podmínek provozu

Obr. 1. Umístění měřicího termočlánku v modelu tavicí pece.

Obr. 2. Přechodová charakteristika soustavy.

Obr. 3. Průběh autokorelační funkce pseudo-náhodného binárního signálu.

Obr. 4. Průběh výkonové spektrální hustoty (model č. 1).

Obr. 5. Průběh průměrné poměrné lineární chyby predikce (model č. 3).

Obr. 6. Průběh průměrné poměrné kvadratické chyby predikce (model č. 3).

ПРИМЕНЕНИЕ СТАТИСТИЧЕСКОЙ ИДЕНТИФИКАЦИИ НА ФИЗИЧЕСКОЙ МОДЕЛИ СТЕКЛОВАРЕННОЙ ПЕЧИ

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На физической модели полностью электрифицированной стекловаренной печи использовали метод статистической идентификации, предоставляющей данные относительно динамического поведения печи для ее необходимого оборудования. Динамические свойства печи рассматривали на основании реакции выходной величины (температура) на скачкообразные изменения входной величины (электрическая мощность). На основании исследования свойств входных сигналов на входе использовали псевдо-случайный бинарный сигнал. С помощью методов статистической идентификации (метод наименьших квадратов и распространенный метод наименьших квадратиков) устанавливались как коэффициенты дифференциальных уравнений так и их порядок, который равняется 3. Приводимые величины были доказаны верификацией окончательных моделей согласно тестам.

На основании оценки математических моделей системы и на основании оценочных критериев можно утверждать, что полученные результаты хорошо отражают динамическое поведение системы. Однако при переводе некоторых моделей с дискретной области в непрерывную были получены отрицательные результаты, и поэтому у всех моделей тестировали их стабильность при помощи программы STABPOL.

Во время измерений были также раскрыты источники погрешностей, которые могут оказать отрицательное влияние на результаты статистической идентификации. Источником важнейших погрешностей оказываются погрешности измерительной установки (напр. температуры). Приводимый недостаток был исключен способом, предлагаемым авторами, на основании которого рекомендуется использование для определения температуры электрическое сопротивление между электродами, измерение которого оказывается гораздо точнее. Пригодным также оказывается присоединение вычислительной машины к модели, с помощью которой можно непрерывно уточнять константы согласно изменяющимся условиям производства.

Рис. 1. Размещение измерительного термоэлемента в модели стекловаренной печи.

Рис. 2. Переходная характеристика системы.

Рис. 3. Ход автокорреляционной функции псевдо-случайного бинарного сигнала.

Рис. 4. Ход мощностной спектральной плотности (модель № 1).

Рис. 5. Ход средней относительной линейной погрешности предикции (модель № 3).

Рис. 6. Ход средней квадратической погрешности предикции (модель № 3).