MECHANICS OF DELAMINATION OF THIN FILMS UNDER THERMAL AND RESIDUAL COMPRESSIVE STRESSES

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An analysis has been made of the delamination of thin films due to uniformly distributed thermal and residual compressive stresses and local thermal stresses arising at the occurence of a hot spot above an interface crack in heated components.

Expressions have been derived for calculation of the strain energy release rate during growth of one-dimensional and circular delamination, and the conditions for start and growth of these delaminations have been analysed.

1. INTRODUCTION

Various components are provided with thin coatings with the aim to improve their properties, among other resistance against high temperatures. As a consequence of coating preparation or of service conditions, steady or transient compressive stresses often act in such coatings. If these stresses reach sufficiently high value, they can cause delamination and spalling of the coating.

Numerous papers have been devoted to the mechanics of delamination of thin films [1–10]. Mostly, homogeneous stress distribution in the film was assumed. In such case, delamination can occur only after the film has buckled. With nonuniform stress distribution, an interface crack can grow even below an unbuckled film. One important example is the so-called hot spot [11]. If heat flows from environment into the component, the coating becomes warmer than the substrate, and compressive stresses arise in it. With good contact film-substrate, the temperature and stress field in the film is homogeneous. If, however, a delaminated area occurs somewhere, the heat flux from the film into the substrate is disturbed, and the film temperature rises here. Nonhomogeneous stress distribution due to a hot spot causes high stress concentration at the edges of an interface crack. If these stresses reach sufficiently high value, the crack can grow provided the energy released during its growth is the same or higher than the energy needed to form new surfaces. At certain magnitude of compressive stress the film buckles, which again modifies the stress distribution and energy released during growth of the delamination. The process is also influenced by the presence of residual stresses and stresses caused by the difference between mean temperatures of the film and substrate.

The aim of this paper is to analyse the growth of interface cracks under various conditions. For the evaluation of crack behaviour, strain energy release rate will be used here. (Another suitable quantity is stress intensity factor. Its evaluation for cracks at bimaterial interfaces, however, is rather complicated (see e.g. [12–14]), and is not necessary for our purpose.) Because real delaminations often have irregular from, energy release rate will be studied here for two limiting cases, one-dimensional and circular delamination. For simplicity, a rigid substrate will be assumed so that only the strain energy of the film and the energy of newly formed surfaces will enter into the energy balance. Further, the interface will be assumed to be plane, the film properties homogeneous and isotropic, and the delamination long compared with the film thickness, so that it will be possible to neglect the changes in stress distribution at the edges of the delamination due to its growth. This growth also results in gradual growth of the area with poor heat transfer from the film into the substrate. An exact analysis should, therefore, involve also time factor. Here, for simplicity, the dimensions of the hot spot will be assumed to be constant, equal to the initial dimensions of the delamination.

2. ONE-DIMENSIONAL DELAMINATION

Consider a delamination of length $2a$ in x-direction and $2b$ in y-direction, with $b >> a$ (Fig. 1). Here, only the conditions for interface crack growth in x-direction in the area remote from lateral edges $\pm b$ will be examined, where the stresses do not depend on y.

2.1 Hot spot in stress-free film

The film above an interface crack can be approximately divided into two sections (Fig. 1b): the inner (1), $2a_0$ long with a hot spot, the mean temperature of which is by ΔT higher than the mean temperature of the surrounding film, and the outer (2), with the same temperature as the film strongly bonded to the substrate. If the delaminated part of the film were removed by an imaginary cut, divided into parts 1 and 2, and released, the dimensions of part 1 would increase with the strain (in x and y directions):

$$\epsilon_{L,0} = \Delta a_0 / a_0 = \alpha \Delta T\,, \tag{1}$$

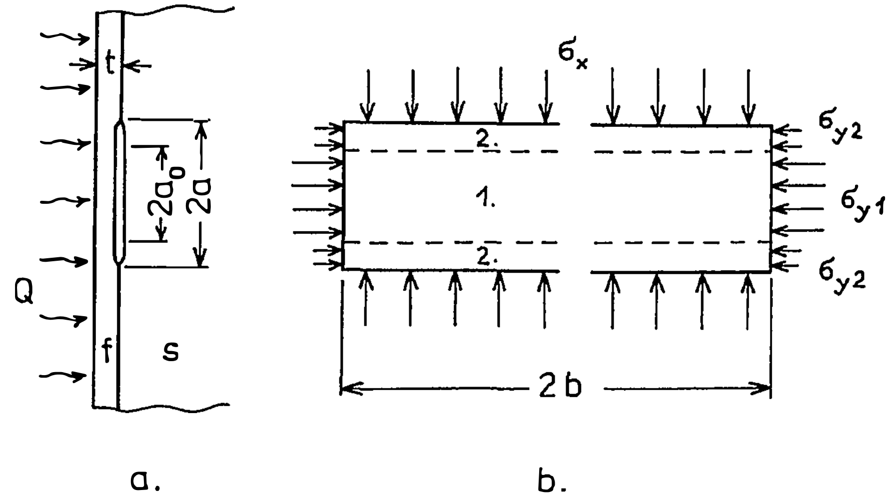

Fig. 1. Delamination with a hot spot. a) schematic diagram, b) stresses in one-dimensional delamination
s – substrate, f – film, t – film thickness, Q – heat flux.

where α is thermal expansion coefficient of the film. The subscript L means local. The dimensions of part 2 would remain unchanged. The mean free strain of parts 1 + 2 in x-direction would be

$$\epsilon_{\mathrm{L}} = \Delta a/a = \alpha \Delta T\,(a_0/a) = \epsilon_{\mathrm{L},0}\,(a_0/a)\ . \tag{2}$$

The film outside the delamination suppresses these elongations, so that compressive stresses σ_x, σ_y arise in parts 1 and 2 (Figs. 1, 2b). Their values will depend on whether the film buckles or not.

a) Strain ϵ_{L} is less than critical

The film is unbuckled, with compressive stresses

$$\sigma_x = \epsilon_{\mathrm{L},0}\,(a_0/a)\,E/\,(1-\nu)\ , \tag{3}$$

$$\sigma_{y1} = \epsilon_{\mathrm{L},0}\,[1-\nu+\nu\,(a_0/a)]\,E/\,(1-\nu)\ , \tag{4}$$

$$\sigma_{y2} = \epsilon_{\mathrm{L},0}\,(a_0/a)\,E\nu/\,(1-\nu)\ , \tag{5}$$

where subscript 1, 2 denotes the part 1 and 2 of the film, respectively. σ_x has the same value in both parts.

The strain energy can be determined as the work of edge stresses done by eliminating the free expansion: first, the stress σ_{y1} compresses the inner part by $2b\epsilon_{\mathrm{L},0}$, then the stress σ_x compresses the parts 1 and 2 by $2a\epsilon_{\mathrm{L}}$. The corresponding work is

$$U = 2Eta_0b\epsilon_{\mathrm{L},0}^2\,[1-\nu+(1+\nu)\,(a_0/a)]\,/\,(1-\nu) \tag{6}$$

The energy release rate during growth of an interface crack is generally

$$\mathcal{G} = \frac{\mathrm{d}U_0}{\mathrm{d}A} - \frac{\mathrm{d}U}{\mathrm{d}A}\ , \tag{7}$$

where $\mathrm{d}U$ is the energy released from the still undelaminated film during the growth of the interface crack by an infinitesimal area $\mathrm{d}A$, and $-\mathrm{d}U$ is the cor-

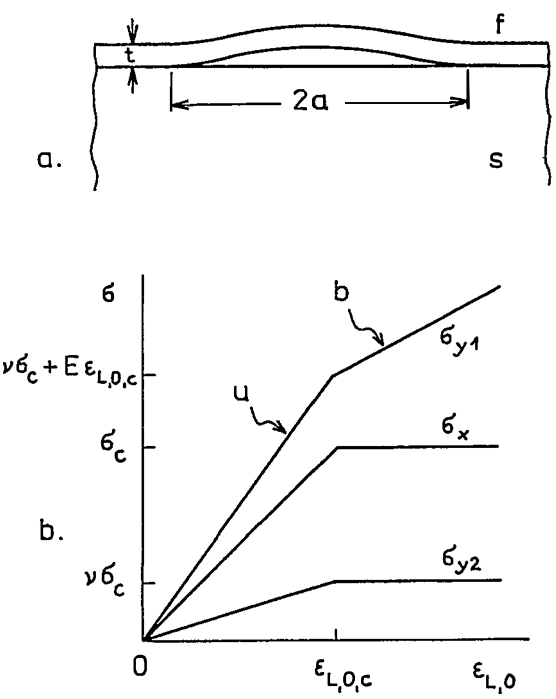

Fig. 2. Buckling of delaminated film due to compressive stresses. a) schematic diagram, b) edge stresses in one-dimensional delamination with a hot spot as a function of local compressive strain $\epsilon_{\mathrm{L},0}$ in part 1. σ_c – critical stress, $\epsilon_{\mathrm{L},0,c}$ – critical strain, E – Young's modulus, ν – Poisson's ratio, u, b – unbuckled and buckled state.

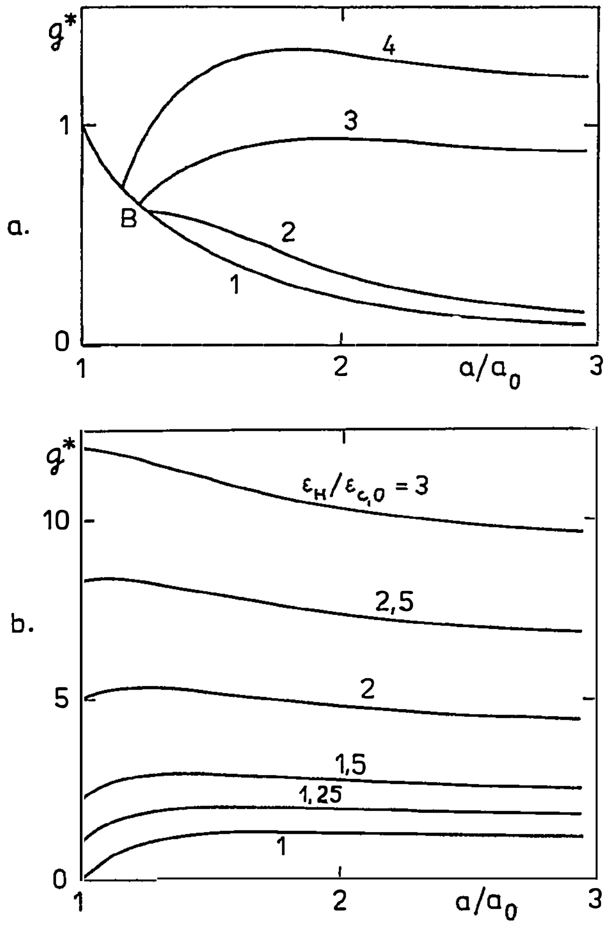

Fig. 3. Energy release rate $\mathcal{G}^*$ as a function of relative length a/a_0 of one-dimensional delamination. a) nonhomogeneous stress due to a hot spot is present; the delamination can grow also in unbuckled state. In B, the film buckles. (1, 2 – no internal stresses are present; curve 2 corresponds to buckled state, 3, 4 – energy release rate increases with the magnitude of internal stresses, b) uniformly prestressed film; the delamination can grow only in buckled state. ϵ_H – homogeneous strain, ϵ_c – critical strain on buckling, a_0 – initial half-length of the delamination, $\mathcal{G}^* = 2\mathcal{G}(1-\nu) / \left[(1+\nu) Et\epsilon_{c,0}^2\right]$.

responding decrease of energy in the delaminated part of the film. So far, no stresses in the film outside the delamination have been assumed, so that $\mathrm{d}U_0 = 0$. The energy release rate pertinent to the crack growth in x-direction is

$$\mathcal{G} = -\frac{1}{2b}\frac{\mathrm{d}U}{2\mathrm{d}a}, \tag{8}$$

where $2a$ corresponds to the symmetrical growth of the delamination in $+x$ and $-x$ directions. Inserting U from (6) gives

$$\mathcal{G} = \epsilon_{L,0}^2 (a_0/a)^2 Et(1+\nu) / [2(1-\nu)] . \tag{9}$$

The course of $\mathcal{G}(a/a_0)$ is depicted by curve 1 in Fig. 3a.

b) Strain ϵ_L is larger than critical

If compressive strain ϵ_L in x-direction exceeds the critical value

$$\epsilon_c = k\left[12(1+\nu)\right]^{-1}(t/a)^2 , \tag{10}$$

the film buckles (Fig. 2a). As it follows from the stability theory of biaxially compressed rectangular plates with clamped edges [15, 16], only the stress σ_x is decisive for buckling when $b >> a$, so that the constant k can be assumed (as a first approximation) to be the same as that for a uniaxially loaded wide beam with clamped edges, $k = \pi^2$

The condition for buckling can be written, with respect to equation (2), in the form

$$\epsilon_{L,0} \gtrsim \epsilon_{L,0,c} = \epsilon_{c,0}(a_0/a) , \tag{11}$$

where $\epsilon_{L,0}$ is given by (1) and $\epsilon_{L,0}$ by (10) with $a = a_0$ and $k = \pi^2$.

With small deflections, the stress σ_x in the buckled film remains approximately constant, equal to the critical stress

$$\sigma_c = kE\left[12(1-\nu^2)\right]^{-1}(t/a)^2 . \tag{12}$$

The stress σ_{y2} in part 2 also remains constant,

$$\sigma_{y2} = \nu\sigma_c . \tag{13}$$

The stress σ_{y1}, however, increases further with increasing $\epsilon_{L,0}$ (or $\alpha\Delta T$), from the value given by (4) for $\epsilon_{L,0} = \epsilon_{L,0,c}$ to

$$\sigma_{y1} = \nu\sigma_c + E\epsilon_{L,0} . \tag{14}$$

The situation is depicted in Fig. 2b. The strain energy, determined again as the work of reaction forces, is

$$U = 2Eta_0b\left\{\epsilon_{L,0}^2 + \left[2\epsilon_{L,0}\epsilon_{c,0}(a_0/a)^2 - \right.\right. \tag{15}$$

$$\left.\left. - \epsilon_{c,0}^2(a_0/a)^3\right](1+\nu)/(1-\nu)\right\} .$$

The energy release rate in buckled state is, according to (8),

$$\mathcal{G} = \left[4\epsilon_{L,0}\epsilon_{c,0}(a_0/a)^3 - 3\epsilon_{c,0}^3(a_0/a)^4\right] \times \tag{16}$$

$$\times Et(1+\nu)/[2(1-\nu)] ;$$

see curve 2 in Fig. 3a.

2.2 Uniformly prestressed film

Films and coatings are often stressed uniformly. Besides residual stress σ_R from film preparation, thermal stress σ_T acts in a film warmer or colder than the substrate. In thin films,

$$\sigma_T = \alpha \Delta T_{fs} E / (1-\nu) \, , \tag{17}$$

where ΔT_{fs} is the difference between mean temperatures of the film and the substrate. The resulting strain is

$$\epsilon_H = \epsilon_R + \epsilon_T = (\sigma_R + \sigma_T)(1-\nu)/E \, . \tag{18}$$

If only uniformly distributed residual or thermal stresses act in the film with a delamination, strain energy can release only in buckled state. The buckling occurs if the total compressive strain ϵ_H exceeds the critical value (10) with $k = \pi^2$. In buckled state, σ_x keeps the critical value σ_c according to (12). The compressive stress σ_y, however, increases further with increasing ϵ_H from σ_c to the value

$$\sigma_y = E\left[\epsilon_H + \epsilon_c \nu / (1-\nu)\right] \, . \tag{19}$$

The strain energy accumulated in the buckled film is

$$U = 2Eta_0 b \left\{ \epsilon_H^2 (a_0/a) + \left[2\epsilon_H \epsilon_{c,0} (a_0/a) - \right. \right. \tag{20}$$

$$\left. \left. - \epsilon_{c,0}^2 (a_0/a)^3 \right] (1+\nu)(1-\nu) \right\} \, .$$

Because the energy of internal stresses, U_0, is accumulated in the film outside the delamination, the strain energy release rate must be determined according to

$$\mathcal{G} = \frac{1}{4b} \frac{\mathrm{d}(U_0 - U)}{\mathrm{d}a} \, . \tag{21}$$

Inserting U from (20) and U_0 from

$$U_0 = 2Etab\epsilon_H^2 / (1-\nu) \tag{22}$$

gives

$$\mathcal{G} = \left[\epsilon_H^2 + 2\epsilon_H \epsilon_{c,0} (a_0/a)^2 - 3\epsilon_{c,0} (a_0/a)^4 \right] \times \tag{23}$$

$$\times \, Et(1+\nu) / [2(1-\nu)] \, .$$

The course is plotted for various ratios $\epsilon_H/\epsilon_{c,0}$ in Fig. 3b.

2.3 Uniformly prestressed film with a hot spot

The stresses σ_x, σ_{y1}, σ_{y2} in unbuckled state are given by equations (3) – (5) with right sides extended by the member $\epsilon_H E/(1-\nu)$, where ϵ_H is given by eqn. (18). The strain energy equals the sum of the internal stresses energy (22) and the energy due to the hot spot (6). The energy of internal stresses, however, does not release during crack growth so that the energy release rate is given by equation (9).

The film buckles if the total compressive strain in x-direction is equal or larger than the critical value ϵ_c, that is if

$$\epsilon_H + \epsilon_{L,0} (a_0/a) \geq \epsilon_{c,0} (a_0/a)^2 \, . \tag{24}$$

(This condition can be fulfilled for various combinations of ϵ_H and $\epsilon_{L,0}$.) After buckling, the stress σ_x remains approximately constant, equal σ_c. The stress σ_{y2} also remains constant, $\sigma_{y2} = \nu\sigma_c + E\epsilon_H$. The stress σ_{y1} increases linearly to the value $\nu\sigma_c + E(\epsilon_H + \epsilon_{L,0})$ for the final strain $\epsilon_H + \epsilon_{L,0}$. The situation is similar like in Fig. 2 with $\epsilon_H + \epsilon_{L,0}$ instead of $\epsilon_{L,0}$.

The strain energy in buckled film can be again determined as the work of reaction forces. The energy release rate is, according to (21)

$$\mathcal{G} = \left[\epsilon_H^2 + 2\epsilon_H \epsilon_{c,0} (a_0/a)^2 + 4\epsilon_{L,0}\epsilon_{c,0} (a_0/a)^3 - \right. \tag{25}$$

$$\left. - 3\epsilon_{c,0}^2 (a_0/a)^4 \right] Et(1+\nu) / [2(1-\nu)] \, .$$

The course is illustrated schematically by curves 3, 4 in Fig. 3a. If the delamination begins to spread only after the film has buckled, the character of $\mathcal{G}$ is similar like that in Fig. 3.

Note. Equation (25) covers both preceding cases. If internal stresses act only (i.e. $\epsilon_{L,0} = 0$), eqn. (25) changes into (23); with the hot spot only ($\epsilon_H = 0$) one obtains equation (16).

2.4 Growth of interface cracks

A delamination beneath a film loaded only by uniformly distributed thermal or residual stresses can grow only if the film has buckled (i.e. if $\epsilon_H \geq \epsilon_{c,0}$), provided the energy release rate $\mathcal{G}_0$, corresponding to the initial crack length a_0, fulfills the following condition

$$\mathcal{G}_0 = \left(\epsilon_H^2 + 2\epsilon_H \epsilon_{c,0} - 3\epsilon_{c,0}^2 \right) \times \tag{26}$$

$$\times \, Et(1+\nu) / [2(1-\nu)] \geq \mathcal{G}_c \, ,$$

where $\mathcal{G}_c$ is the critical energy release rate (specific fracture energy) of the interface. The crack growth will continue if $\mathcal{G}(a) \geq \mathcal{G}_0$. Expressing $\mathcal{G}(a)$ from (23) and denoting as ϵ_H^* the minimum compressive strain, at which the condition (26) for start of a delamination is fulfilled, gives the following condition for spontaneous delamination growth:

$$\epsilon_H^* / \epsilon_{c,0} \leq 1.5 \left[1 + (a_0/a)^2 \right] \, . \tag{27}$$

According to the ratio $\epsilon_H^*/\epsilon_{c,0}$ three various modes of crack growth[1] can occur (Fig. 3b):

[1] This behaviour is similar like that of one-dimensional delamination in a component compressed uniaxially, see [4, 8].

a) $1 \leq \epsilon^*_{\mathrm{H}}/\epsilon_{\mathrm{c},0} < 1.5$
Energy release rate $\mathcal{G}(a)$ is for any crack length a higher than the starting value $\mathcal{G}_0$. The delamination will spread in an unstable way, hypothetically to infinite length, in fact untill spalling occurs.

b) $1.5 \leq \epsilon^*_{\mathrm{H}}/\epsilon_{\mathrm{c},0} \leq 3$
Crack growth is unstable at its beginning but it stops after some time, and could continue only in a stable way by increasing ϵ_{H}. The final crack length depends on the ratio and on energy dissipation during crack growth.

c) $3 < \epsilon^*_{\mathrm{H}}/\epsilon_{\mathrm{c},0}$
The delamination can spread only in a stable way, i.e. by continuous increasing of ϵ_{H} (e.g. by heating the film).

If, for some reason, the delamination starts growing with strain $\epsilon_{\mathrm{H}} > \epsilon^*_{\mathrm{H}}$, its further growth can differ from the aforesaid.

In presence of a hot spot, an interface crack can grow under unbuckled film provided the following condition is fulfilled:

$$\mathcal{G}_0 = \epsilon^2_{\mathrm{L},0} Et\,(1+\nu)\,/\,[2\,(1-\nu)] \geq \mathcal{G}_{\mathrm{c}}\,, \tag{28}$$

which follows from (9) for $a = a_0$. If we consider e.g. a film with $\mathcal{G}_{\mathrm{c}} = 20\ \mathrm{Jm}^{-1}$, $E = 300$ GPa, $\nu = 0.2$, $t = 0.2$ mm, and $\alpha = 7.7 \times 10^{-6}\ \mathrm{K}^{-1}$, we can see with respect to eqn. (1) that an interface crack can grow if the mean temperature of the film above the interface crack is only by $\Delta T = 86.6$°C higher than the mean temperature of the surrounding film. Moreover, because it is not necessary to fulfil the condition for buckling, relatively small cracks can propagate in this way. According to (28), the initial value of energy release rate does not depend on crack length. The presented theory, however, is valid only for sufficiently high ratios a/t, where the change of stress distribution at the crack tip during crack growth can be neglected.

If no internal stresses are present in the film, $\epsilon_{\mathrm{H}} = 0$, energy release rate decreases monotonically with the delamination length, from $\mathcal{G} = \mathcal{G}_0$ at $a = a_0$ to zero at a approaching to infinity (curve 1 in Fig. 3a). An interface crack can thus begin to grow only if $\mathcal{G}_0 > \mathcal{G}_{\mathrm{c}}$, and stops as soon as $\mathcal{G}(a)$ has decreased below $\mathcal{G}_{\mathrm{c}}$. However, during delamination growth, the hot spot sometimes grows gradually, too. In such case, the ratio a_0/a does not decrease, and the same holds for energy release rate. As a criterion for failure, $\mathcal{G}_0 = \mathcal{G}_{\mathrm{c}}$ must be therefore used. In detailed analysis, the time dependence of the film temperature during the delamination process should be considered.

If the condition (24) is fulfilled, the film buckles.In buckled state, the energy of uniformly distributed residual and thermal stresses begins to release, and $\mathcal{G}$ does not decrease to zero with increasing a, but to the asymptotic value

$$\mathcal{G}_\infty = \epsilon^2_{\mathrm{H}} Et\,(1+\nu)\,/\,[2\,(1-\nu)]\,; \tag{29}$$

see curves 2–4 in Fig. 3a. For $\mathcal{G}_\infty \geq \mathcal{G}_{\mathrm{c}}$ the interface crack could (hypothetically) grow to infinite length. In fact, the process will stop after some time due to spalling or other effects.

The course of energy release rate depends on the relative values ϵ_{H}, $\epsilon_{\mathrm{L},0}$ and $\epsilon_{\mathrm{c},0}$. For $\epsilon_{\mathrm{L},0}+\epsilon_{\mathrm{H}}/3 > \epsilon_{\mathrm{c},0}$, the function $\mathcal{G}(a)$ decreases monotonically from the beginning. For $\epsilon_{\mathrm{L},0}+\epsilon_{\mathrm{H}}/3 = \epsilon_{\mathrm{c},0}$, $\mathcal{G}(a)$ has a maximum at $a = a_0$, and decreases monotonically for $a > a_0$. For $\epsilon_{\mathrm{L},0} + \epsilon_{\mathrm{H}}/3 < \epsilon_{\mathrm{c},0}$, the function $\mathcal{G}(a)$ increases at first, reaches a maximum at certain value $a^* > a_0$ and decreases. For

$$\epsilon_{\mathrm{H}} + \epsilon_{\mathrm{L},0} > \epsilon_{\mathrm{c},0}\,, \tag{30}$$

the film buckles before crack growth, or simultaneously with it. (The first, second, and part of the third from the aforesaid cases belong here.) If the condition (24) is fulfilled but the condition (30) not, buckling occurs later. The corresponding critical delamination length a_{c} can be found by solving the quadratic equation (24).

3. CIRCULAR DELAMINATION

The circular delamination has been devoted greater attention in the literature. The analysis of delamination of uniformly compressed film can be found in [1], the solution for the hot spot can be obtained by adapting the solution for the case with a film compressed above the delamination by an indentation imprint [2]. Therefore, only the main results will be summarised here.

3.1 Hot spot in stress-free film

Consider a circular delamination of radius a. In the inner part of the delaminated film there is a hot spot with radius a_0 and mean temperature by ΔT higher than the mean temperature of the surrounding film (Fig. 1a). The film outside the delamination is strongly bonded to the substrate, and prevents the hotter part from increasing its radius. As a result, stresses arise in this area, which are, moreover, distributed nonuniformly when $a > a_0$.

The distribution of stresses and strains in the delaminated part of the film is similar like that in a ring with radius a shrink-fitted on a disc of radius a_0 with the radius allowance $\Delta a_0 = a_0 \alpha \Delta T$, and then compressed by radial forces so that the outer radius becomes again a. The pertinent solution can be found in [16, 17] or in [2]. Notice that the relative change of radius a is now

$$\epsilon_{\mathrm{L}} = \Delta a/a = \alpha \Delta T\,(a_0/a)^2 = \epsilon_{\mathrm{L},0}\,(a_0/a)^2\,. \tag{31}$$

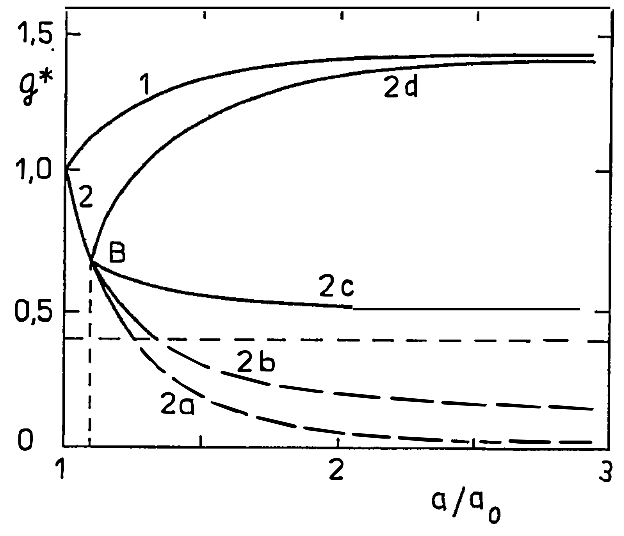

Fig. 4. Energy release rate $\mathcal{G}^*$ as a function of relative radius a/a_0 of a circular delamination, 1 – uniformly prestressed film; the delamination can grow only in buckled state, 2 – nonhomogeneous stress due to a hot spot is present; the delamination can grow also in unbuckled state. In B, the film buckles. (2a – only hot spot stresses are present in the film, 2b, c, d – energy release rate increases with the amount of internal stresses), $\mathcal{G}^* = 2\mathcal{G}\,(1-\nu)\,/\,\left[(1+\nu)\,Et\epsilon_{c,0}^2\right]$.

The strain energy in the film above the interface crack can be determined as the sum of the work U_1 of reaction forces acting on its edge during the compression by Δa and the energy U_2 of internal stresses resulting from the nonhomogeneous stress distribution if the hot spot radius a_0 is smaller than the delamination radius a,

$$U = U_1 + U_2\,. \tag{32}$$

The energy of internal stresses is [2]

$$U_2 = \pi Et\epsilon_{L,0}^2 a_0^2 \left[1-(a_0/a)^2\right]/2\,. \tag{33}$$

The work of radial forces performed during the compression depends on whether the film buckles or not.

a) Strain ϵ_L is less than critical

The film is unbuckled, and radial stress is related to the strain ϵ by

$$\sigma = \epsilon E/(1-\nu)\,. \tag{34}$$

The work of these stresses is

$$U_1 = \pi Et\epsilon_{L,0}^2 a_0^2 (a_0/a)^2/(1-\nu)\,. \tag{35}$$

The energy release rate during growth of an interface crack is again given by equation (7). Now, with $U_0 =$ 0 (no stress in the film outside the delamination is assumed) and $dA = 2\pi a da$, one obtains

$$\mathcal{G} = \epsilon_{L,0}^2 (a_0/a)^4\, Et\,(1+\nu)/[2(1-\nu)]\,, \tag{36}$$

where a_0 is initial and a instantaneous radius of the delamination. The course $\mathcal{G}$ is depicted in Fig. 4 (curve 2, 2a).

b) Strain ϵ_L is larger than critical

If ϵ_L exceeds the critical value (10) with $k = 14.68$ (circular plate with clamped edge [15, 16], the film buckles. In buckled state, the radial compressive stress σ at the edge of delamination depends on the relative radial displacement Δa by [1]

$$(\sigma/\sigma_c) - 1 = \beta[(\Delta a/\Delta a_c) - 1]\,, \tag{37}$$

where σ_c is the critical stress, given by equation (34) with $\epsilon = \epsilon_c$, a_c is the critical contraction of radius a, and

$$\beta = \left[1 + 1.108\,(1-\nu)^{-1}\right]^{-1} \tag{38}$$

is a constant[2]. The course of radial stress is similar to that of σ_{y1} in Fig. 2.

The work U_1 of radial stresses done during compression of the disc with radius a by Δa is

$$U_1 = \pi Eta_0^2 (a_0/a)^2 \left[\epsilon_{L,0}^2 - (1-\beta)(\epsilon_{L,0}-\epsilon_{c,0})^2\right]/(1-\nu)\,; \tag{39}$$

The corresponding energy release rate is with respect to (7), (32), (33) and (39)

$$\mathcal{G} = (a_0/a)^4 \left[(1+\nu)\,\epsilon_{L,0}^2/2 - (1-\beta)(\epsilon_{L,0}-\epsilon_{c,0})^2\right] Et/(1-\nu)\,; \tag{40}$$

see curve 2a in Fig. 4.

3.2 Uniformly prestressed film

The situation in a uniformly compressed film with a circular delamination has been throughly investigated in [1, 2]. The stress in an unbuckled film is distributed uniformly everywhere so that energy release rate is zero. If $\epsilon_H > \epsilon_c$, where ϵ_H is given by eqn (18), the film buckles, which is accompanied by partial stress relaxation. As a result, energy releases during the delamination process as the initally unbuckled part of the film gets buckled. The energy release rate is

$$\mathcal{G} = (1-\beta)\left[\epsilon_H^2 - \epsilon_{c,0}^2 (a_0/a)^4\right] Et/(1-\nu)\,; \tag{41}$$

see curve 1 in Fig. 4.

[2] Formula (38) was originally written [1] as $\beta = [1+1.207(1+\nu)]^{-1}$ but was later corrected by J. W. Hutchinson [18].

3.3 Uniformly prestressed film with a hot spot

If an interface crack occurs in a heated component, both kinds of stress discussed earlier act together: The situation is similar like that in a uniformly prestressed film with local stress induced by indentation in the delaminated area [2]. In unbuckled state, the energy of uniformly distributed stresses does not release during crack growth so that $\mathcal{G}$ is given by eqn (36). If the total compressive strain exceeds the critical value, i.e. if

$$\epsilon = \epsilon_{\mathrm{H}} + \epsilon_{\mathrm{L},0}\,(a_0/a)^2 > \epsilon_{\mathrm{c},0}\,(a_0/a)\ , \tag{42}$$

the film buckles. In the released energy, the component corresponding to the hot spot reduces, and the energy of internal stresses begins to apply. The energy release rate is

$$\mathcal{G} = \Big\{(1-\beta)\,\epsilon_{\mathrm{H}}^2 + (a_0/a)^4\Big[(1+\nu)\,\epsilon_{\mathrm{L},0}^2/2- \tag{43}$$

$$-(1-\beta)\,(\epsilon_{\mathrm{L},0}-\epsilon_{\mathrm{c},0})^2\Big]\Big\}\,Et/(1-\nu)\ ,$$

where ϵ_{H} given by eqn (18), represents the homogeneous component of the total strain ϵ. The course of $\mathcal{G}\,(a_0/a)$ is plotted for various ratios $\epsilon_{\mathrm{H}}/\epsilon_{\mathrm{c},0}$ in Fig. 4 (curves 2b, c, d).

Note. Equation (43) holds also for unbuckled film (with $\beta = 1$), and embraces all cases from Section 3.1 and 3.2.

3.4 Growth of interface cracks

If only uniformly distributed stress acts, an interface crack can grow only after the film has buckled, i.e. when $\epsilon_{\mathrm{H}} \geq \epsilon_{\mathrm{c}}$, and if

$$\mathcal{G}_0 = (1-\beta)\left(\epsilon_{\mathrm{H}}^2 - \epsilon_{\mathrm{c},0}^2\right) Et/(1-\nu) \geq \mathcal{G}_{\mathrm{c}}\,, \tag{44}$$

where $\mathcal{G}_{\mathrm{c}}$ is the critical energy release rate of the interface. Because $\mathcal{G}(a)$ grows monotonically (see eqn. (41) and curve 1 in Fig. 4) with asymptotic value

$$\mathcal{G}_\infty = (1-\beta)\,\epsilon_{\mathrm{H}}^2 Et/(1-\nu)\ , \tag{45}$$

the crack growth will be unstable. Theoretically, the whole film could delaminate. After some growth of interface cracks, however, the film usually begins to spall [1, 3, 11].

With a hot spot above the interface crack, the delamination can grow also in unbuckled state; only the condition (28) must be fulfilled. The crack growth in unbuckled state has similar character like with one-dimensional delamination, the energy release rate, however, decreases faster. If the condition (42) is fulfilled, the film buckles. Now, also the energy of uniformly distributed stresses applies, and the energy release rate is given by eqn. (41). Characteristic course of $\mathcal{G}(a)$ is depicted by curves 2b, c, d in Fig. 4. The asymptotic value $\mathcal{G}_\infty$ is again given by equation (45). If $\mathcal{G}_\infty \geq \mathcal{G}_{\mathrm{c}}$ the crack can grow (hypothetically) to infinite length, if $\mathcal{G}_\infty < \mathcal{G}_{\mathrm{c}}$ the crack stops after some time depending on the rate of energy dissipation during its growth.

Note. The considerations from Section 2.4 and 3.4 are, in general, valid also for subcritical crack growth promoted by corrosive action of the environment, provided $\mathcal{G}_0$ is higher than the fatigue limit $\mathcal{G}_{\mathrm{scc}}$. In such cases, however, the dependence of crack velocity on $\mathcal{G}$ and on environment must be taken into account.

4. SUMMARY

The conditions and character of delamination growth in thin films have been theoretically investigated for one-dimensional and circular delaminations by means of energy release rate.

If only uniformly distributed residual or thermal compressive stresses act in the film, an interface crack can grow only after the film has buckled. The growth of a circular delamination proceeds unstably, and usually ends with spalling of the film. A very elongated (one-dimensional) delamination buckles at lower stresses than the circular one, and the crack spreads stably or unstably according to the relation between the actual and critical strain.

The growth of an interface crack can be promoted markedly by the action of local compressive stresses, which arise in heated components as a result of temperature increase in the film above the delamination (hot spot). These local stresses act together with uniformly distributed stresses, creating thus more favourable conditions for buckling. Having sufficient value, the local stresses alone can cause growth of a delamination under an unbuckled film. In this case, the energy release rate decreases monotonically. If the film buckles, uniformly distributed stresses begin to apply, and the crack growth can be stable or unstable with a circular as well as one-dimensional delamination.

The formulae for the energy release rate, given in the paper, enable to analyse start and growth of interface cracks, and to estimate the allowable defect size in films with the stress known, and vice versa. Because of considerable influence of local stresses due to, e.g. hot spots, an exact analysis must respect the real temperature distribution and development in the component.

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MECHANIKA DELAMINACE TENKÝCH VRSTEV PŮSOBENÍM TEPLOTNÍCH A ZBYTKOVÝCH TLAKOVÝCH NAPĚTÍ

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V práci jsou analyzovány podmínky a průběh delaminace tenkých vrstev při různých způsobech zatížení a tvarech delaminované oblasti. Působí-li ve filmu pouze rovnoměrně rozložené zbytkové nebo teplotní tlakové napětí, může dojít k růstu trhliny na rozhraní pouze po vyboulení filmu. Šíření delaminace kruhového tvaru probíhá nestabilně a zpravidla končí praskáním a odlupováním částí povlaku. U značně protáhlé (jednorozměrné) delaminace dochází k boulení při nižším napětí než u kruhové delaminace, a trhlina se může šířit nestabilně nebo stabilně v závislosti na poměru skutečného a kritického poměrného stlačení.

Růst trhliny na rozhraní může být výrazně usnadněn působením lokálních tlakových napětí, která vznikají v důsledku zvýšení teploty filmu nad delaminací u ohřívaných součástí (tzv. horká skvrna). Tato lokální napětí se sčítají s dalšími, rovnoměrně rozloženými napětími a vytvářejí tak příznivější podmínky pro boulení. Při dostatečné velikosti však mohou lokální napětí samotná vést k šíření delaminace i u nevybouleného povlaku. V tomto případě rychlost uvolňování energie monotónně klesá. Dojde-li však k vyboulení povlaku, začnou se uplatňovat i ostatní složky napětí a proces delaminace může být buď stabilní nebo nestabilní, a to u jednorozměrné nebo i kruhové delaminace.

Vztahy pro výpočet rychlosti uvolňování energie, uvedené v článku, umožňují podrobnější analýzu startu a růstu trhliny na rozhraní, a stanovení přípustné velikosti delaminace u povlaků s určitým vlastním pnutím a naopak. S ohledem na velký vliv lokálních napětí v místech s horkou skvrnou musí analýza napjatosti v těchto případech vycházet z analýzy teplotního pole v součásti.

Obr. 1. Delaminace s horkou skvrnou. a) schéma, b) napětí ve filmu při jednorozměrné delaminaci s – substrát, f – film, t – tloušťka filmu, Q – tepelný tok.

Obr. 2. Boulení delaminovaného filmu působením tlakových napětí. a) schéma, b) napětí na okraji jednorozměrné delaminace s horkou skvrnou v závislosti na lokálním poměrném stlačení $\epsilon_{L,0}$ v části 1. σ_c – kritické napětí, $\epsilon_{L,0,c}$ – kritické poměrné stlačení, E – Youngův modul, ν – Poissonovo číslo.

Obr. 3. Rychlost uvolňování energie $\mathcal{G}^*$ v závislosti na relativní délce a/a_0 jednorozměrné delaminace. a) jsou přítomna nehomogenní napětí vyvolaná horkou skvrnou; delaminace může růst i v nevybouleném stavu. V bodě B začíná docházet k boulení filmu. 1, 2 – nejsou přítomna žádná vnitřní napětí; křivka 2 odpovídá vyboulenému stavu, 3, 4 – rychlost uvolňování energie roste s velikostí vnitřního napětí. b) rovnoměrně stlačený film; delaminace může růst pouze ve vybouleném stavu. ϵ_H – homogenní přetvoření, ϵ_c – kritické poměrné stlačení při boulení, a_0 – počáteční poloviční délka delaminace, $\mathcal{G}^* = 2\mathcal{G}(1-\nu) / \left[(1+\nu) Et\epsilon_{c,0}^2\right]$.

Obr. 4. Rychlost uvolňování energie $\mathcal{G}^*$ v závislosti na relativním poloměru a/a_0 kruhové delaminace. 1 – rovnoměrně stlačený film; delaminace může růst pouze ve vybouleném stavu, 2 – jsou přítomna nehomogenní napětí vyvolaná horkou skvrnou; delaminace může růst i v nevybouleném stavu. V bodě B začíná docházet k boulení filmu. (2a – ve filmu je přítomna pouze horká skvrna, 2b, c, d – rychlost uvolňování energie roste s velikostí vnitřních napětí), $\mathcal{G}^* = 2\mathcal{G}(1-\nu) / \left[(1+\nu) Et\epsilon_{c,0}^2\right]$.