TRANSMISSION CHARACTERISTICS OF SINGLE-MODE FIBERS

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This paper describes the using of mathematical algorithms to compute cut-off wavelength, chromatic dispersion, splice losses and micro/macro bending losses of arbitrary circuit symmetric SM structures for a chosen wavelength. The aim is to determine the influence of the fibre design on the observed transmission charactristics, when the dependence of refractive index on wavelength is involved by Sellmeier's coefficients. The appropriate parameters of fibre design are discussed.

1. INTRODUCTION

Single-mode (SM) fibres are considered to become the main transmission medium for future digital telecomunication systems. Their advantages are low attenuation at higher wavelengths ($\alpha < 0.4$ dB/km at 1.3 μ m and $\alpha < 0.2$ dB/km at 1.55 μ m) and very high transmission capacity due to low chromatic dispersion D_0 . These two parameters tohether with cut-off wavelength λ_c are strongly dependent on the structural design of the fiber, especially on the geometry of core and cladding and on the ratio of their refractive indeces.

Theoretical research has not focused sofar on complete description of characteristic, but on parameters involved merely. Karásek [5] deals with the triangular core design. Jeuhonome [12] discusses stepindex design, where materials dispersion is described by approximation formulae only. Neumann [10] does not deal with relationship of dispersion and fibre structure. Sammut [4] and Love [5] present only approximation formulae to compute cut-off wavelength. Article [13] presents FFD structure from the viewpoint of attenuation and bending losses depending on bending radius. Design parameters are included in summarizing structure criteria, but it was a necessary to know the dependences for used technology in detail.

Therefore it was necessary to compare fibres structure and to greate the possibilities for taking into accout the impact of material composition of fibre and defects of fibre profile caused by technology. This methodological concept may be used for classical fibre design or sensor design or fibre with other dopants. The concrete aim of our work was to study the influence of changes in structural design on transmission parameters w_{∞}/\bar{w} , D_0 and λ_c . For this purpose, mathematical computing of fiber parameters has been used. All programs were written in FORTRAN 77 and calculated by IBM AT computer. This mathematical algorithm can be used to compute the transmission characteristics of arbitrary circuit symmetric structures for a chosen wavelength. The dependence of refractive index on wavelength, which is characteristic for the given composition, is taken into account by means of Sellmeier's coefficients [2,7].

2. EXPERIMENTAL

2.1. Fundamental equations used for computing

The algorithm for computing chromatic dispersion D_0 according to Karásek [5] was modified and extended to involve all variations of MCVD technology available at the equipment in operation in our institute [9]. To calculate the propagation constant for mode it is sufficient to solve the simplified form of the scalar wave equation:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}\psi}{\mathrm{d}R} + V^2 \times \left[\bar{\beta} - \mathrm{f}\left(R\right)\right] \times \psi = 0, \qquad (1)$$

where ψ is radial wave function, R = r/a normalised radial coordinate, $k = 2\pi/\lambda$ is free space wave number, $\bar{\beta}$ represents the dominant mode propagation constant in its normalised form and $V = ka\sqrt{2}\Delta$ is normalised frequency; the relationship of parameters a, r and Δ is obvious from Fig. 1.

In equation (1), the differenciations are replaced by central differences and this leads to a systems of linear algebraical equations, which can be solved by iterative process. As a result, we get the normalized propagation constant $\bar{\beta}_{01}$, which is used to calculate chromatic dispersion [5]. Chromatic dispersion per unit of radiation source spectral width is called coefficient of chromatic dispersion D; it is expressed [10,5] as

$$D = \frac{\partial \tau}{\partial \lambda} = \frac{\mathrm{d}\tau}{\mathrm{d}\lambda} = -\frac{L}{2\pi c} \left(2\lambda \frac{\mathrm{d}\beta}{\mathrm{d}\lambda} - \lambda^2 \frac{\mathrm{d}^2\beta}{\mathrm{d}\lambda^2} \right) , \qquad (2)$$

where τ is group delay time, L is fiber length, c is velocity of light in vacuum. Using the same algorithm, it was possible to calculate the spot size of electromagnetic field ψ , which is important for evaluating



Fig. 1. Schematic view of SM fiber. Δ_1 means: $\Delta_1 = (n_1 - n_c)100/n_c$; Δ_2 means: $\Delta_2 = (n_2 - n_c)100/n_c$; n_c depicts pure SiO₂.



Fig. 2. Base design types of SM fibers: A) depressed cladding $\Delta_1 = 0.30\%$, $\Delta_2 = -0.04\%$; B) matched cladding $\Delta_1 = 0.34\%$; C) cladding doped with Ge $\Delta_1 = 0.34\%$, $\Delta_2 = 0.06\%$; D) fully fluorine doped (FFD) $\Delta_1 = 0.00\%$, $\Delta_2 = -0.034\%$.

splicing losses and micro/macro bending losses [11]. For splice loss \bar{w} :

$$\bar{w} = 2 \int_0^\infty \psi^2 r \mathrm{d}r / \int_0^\infty \left(\frac{\mathrm{d}\psi}{\mathrm{d}r}\right)^2 r \mathrm{d}r \tag{3}$$

and for micro/macro bending losses w_{∞} :

$$w_{\infty} = \lambda / \left[2\pi n_1 \left(\beta - k n_2 \right) \right], \qquad (4)$$

where n_1 and n_2 are refractive indices of core and cladding, respectively.

If the ratio w_{∞}/\bar{w} is close to 1, splice losses and micro/macro bending losses will be small [5].

For single-mode operation, cut-off wavelength λ_c is one of important parameters. To investigate the



Fig. 3. Schematic view of the central dip (The equations (7) and (8) hold for $0 \le R < 1$ and $0 \le R \le w_d/a$, respectively.

A: Description by eq. (7);

B: Description by eq. (8).

dependence of λ_c on geometrical fiber structure (design), calculations were made using a program based on the following equation:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}\psi}{\mathrm{d}R} + \left(V_{\mathrm{c}}^2\left[1 - \mathrm{f}\left(R\right)\right] - \frac{1}{R^2}\right)\psi,\qquad(5)$$

The dependence of the radial wave function ψ on R for second mode can be described [3] by

$$\psi(R) = R \exp\left(-\left(R^2 - 1\right)\right) \,. \tag{6}$$

The iterative calculations were made using the divided intervals methods, where derivations were replaced by central differences.

2.2. Influence of fiber design on transmission parameters

For the study of fiber design on transmissin parameters, four basic structure types were chosen (Fig. 2).

The geometry of fiber design can be mathematically expressed as normalized profile refractive index function [4, 6, 9, 10]. With regard to MCVD technology, it was necessary to involve the existence of the so-called central dip (Fig. 3). For this purpose [4, 9], the following expressions were used:

$$f(R) = \gamma (1-R)^{\rho}$$
(7)

$$f(R) = 1 - (1 - w_d/a)R,$$
 (8)

where ρ is power-law coefficient characterizing the width of the central dip, w_d is the width of the central dip and γ is the relative depth of the central dip (Fig. 3).



Fig. 4. The influence of changing refractive-index of core (Δ_1) on zero chromatic dispersion (D_0) at wavelength 1300 nm. Parameters of fibers:

+ $\Delta_1 = 0.4\%$; $\Delta_2 = -0.04\%$; $b = 30 \ \mu m$;

- $\Delta_1 = 0.3\%; \ \Delta_2 = -0.04\%; \ b = 30\mu m;$
- $\Delta_1 = 0.2\%; \ \Delta_2 = -0.04\%; \ b = 30\mu m.$



Fig. 5. The influence of changing refractive-index of core $(\Delta_1) \text{ on } w_{\infty}/\bar{w}$. Parameters of fibers: $+ \Delta_1 = 0.4\%; \ \Delta_2 = -0.04\%; \ b = 30 \ \mu m;$ $\bullet \ \Delta_1 = 0.3\%; \ \Delta_2 = -0.04\%; \ b = 30 \ \mu m;$ $\bullet \ \Delta_1 = 0.2\%; \ \Delta_2 = -0.04\%; \ b = 30 \ \mu m.$

For single mode transmission chosen at 1.3 μ m (low attenuation), the following conditions would be prefered since λ_c must be lower than chosen wave-Silikáty č. 1, 1992



Fig. 6. The influence of dip width on D_0 computed by straight-line approximation for which the initial value $D_0 = 0.0 \text{ ps/km.nm} (\lambda = 1300 \text{ nm}); \Delta_1 = 0.384\%,$ $\Delta_2 = -0.075\%, a = 3.96 \mu m, b = 33.3 \mu m.$

length for transmission and zero dispersion characteristic enables to increase transmission rapid in Mbyt/s. The ratio w_{∞}/\bar{w} allows to predict losses under coupling of fibre and bend. It should be equaled one to be optimal:

$$\lambda_{\rm c} = 1.2 \pm 0.1 \,\mu{
m m}$$

 $D_0 = 0.0 \pm 2 \,{
m ps/km.nm}$
 $w_{\infty}/\bar{w} \equiv 1$

Fig. 4 shows the obtained dependence of zero chromatic dispersion D_0 on core radius a for three different refractive index differences Δ_1 . Zero chromatic dispersion at 1.3 μ m is reached for values of $a \div 4.8 \mu$ m.

Fig. 5 shows the effect of index difference Δ_1 on the ratio w_{∞}/\bar{w} . Structures with small values of Δ_1 are very sensitive to micro/macro-bending losses for the radius $a < 4.0 \ \mu\text{m}$. If Δ_1 increases by about 50% micro/macro-bending losses decrease by about 59% and ratio w_{∞}/\bar{w} is smaller by 45%. The higher the radius *a*, the lower is the sensitivity of the fiber to micro/macro-bending losses and splice losses for the given structure.

Fig. 6 depicts the influence of dip width w_d on chromatic dispersion of a structure, which for $w_d = 0$ μ m has $D_0 = 0$. It shows that for low values of w_d (up to 0.9) the influence of the dop is negligible, because the initial increase of chromatic dispersion lies within the interval ± 0.05 ps/km.nm.



Fig. 7. The dependence of D_0 under changing radius <u>a</u> and for different values Δ_2 ; Parameters of fibers: • $\Delta_1 = 0.20\%$; $\Delta_2 = -0.04\%$; $b = 30 \,\mu m$; + $\Delta_1 = 0.20\%$; $\Delta_2 = -0.14\%$; $b = 30 \,\mu m$.

Fig. 7 indicates that only more pronounced changes of Δ_2 (0.04 = 0.14%) influence the dependence of D_0 on a; slight modifications (Δ_2 from 0.04 to 0.02) have no significant effect. The same applies to the relationship $w_{\infty}/\bar{w} - a$, as can be seen from Fig. 8. This figure also demonstrates, that the structure characterized by $\Delta_1 = 0.2\%$ and $\Delta_2 = -0.14\%$ leads to high values of w_{∞}/\bar{w} , which indicates, that this structure is very sensitive to micro/macrobending losses.

As for the influence of cladding and core radius (ratio b/a in Fig. 1), it was found that the effect of b/a on the ratio is more prominent for lower values of Δ_1 (Fig. 9). If we consider structures with Δ_1 0.3 - 0.4%, then the change of b/a from 1 to 3 has no significant consequences.

Fig. 10 compares the dependence of w_{∞}/\bar{w} on core radius *a* for the four chosen basic structures shown in Fig. 2.

Fig. 11 demonstrates how cut-off wavelength λ_c depends on the relative depth γ of the central dip for different values of coefficient ρ characterizing the width of the dip. The graph in Fig. 12 indicates the influence of changing α -profile (being explained in [10] in detail) and Δ_1 on the position of λ_c .

3. DISCUSSION AND CONCLUSION

The developed programs allow to study the main transmission characteristics of single-mode fibers with circular core and any arbitrary index profile.



Fig. 8. The value of the relation w_{∞}/\bar{w} as function of the depressed cladding depth (Satisfactory relation of w_{∞}/\bar{w} is achieved for structure with $\Delta_1 = 0.3\%$ ar radius $\underline{a} \in < 4.4; 5.0 \,\mu\text{m} >$). Parameters of fibers:

- ▲ $\Delta_1 = 0.2\%$; $\Delta_2 = -0.14\%$; $b = 30 \, \mu m$;
- + $\Delta_1 = 0.3\%$; $\Delta_2 = -0.04\%$; $b = 30\mu m$;
- $\Delta_1 = 0.3\%$; $\Delta_2 = -0.03\%$; $b = 30 \, \mu m$;
- $\circ \Delta_1 = 0.3\%; \ \Delta_2 = -0.02\%; \ b = 30 \mu m.$



Fig. 9. The influence of b/a on the course of w_{∞}/\bar{w} . Parameters of fibers:

+ $\Delta_1 = 0.15\%$; $\Delta_2 = -0.04\%$; $a = 4 \, \mu m_i$

▲ $\Delta_1 = 0.20\%$; $\Delta_2 = -0.04\%$; $a = 4\mu m$;

- $\Delta_1 = 0.30\%; \ \Delta_2 = -0.04\%; \ a = 4 \, \mu m;$
- $\Delta_1 = 0.40\%; \ \Delta_2 = -0.04\%; \ a = 4 \, \mu m.$

The calculations also involve the effect of material dispersion. By determining the coefficients of Sellmeier's equation, that is by providing the refractive



Fig. 10. The dependence of w_{∞}/\bar{w} values for basic design types.



Fig. 11. The influence of relative depth of central dip γ on cut-off wavelength λ_c . Parameters of fibre: $\Delta_1 =$ 0.384%; $\Delta_2 = -0.075\%$; $a = 2.5 \,\mu\text{m}$; $b = 33.3 \,\mu\text{m}$. Relative broad of central dip: I: $\rho = 50$; II: $\rho = 5$; III: $\rho = 1$.

index-wavelength dependence of the given dopant, it is possible to modify the calculations for any dopant.

Comparison of zero chromatic dispersion D_0 for the studied structures (Fig. 13) demonstrates, that their dispersion properties are similar and in the most interesting region $(a \div 4 \ \mu m)$ they don't exhibit any significant differences.

The fully fluorine-doped structure (FFD) is characterized by higher micro/macro bending losses with growing core radius. This structure has an advantage in achieving very small attenuation values, but uts technological realization by MCVD technique requires very high temperatures, which in its turn could lead to deformation of geometry along the preform and to undesirable changes in index profile. The



Fig. 12. The influence of changing α -power law design of core and Δ_1 on cut-off wavelength position. <u>A</u>: α profil versus λ_c ($\Delta_1 = 0.30\%$; $\Delta_2 = -0.04\%$; a =

 $\underline{A}: \ \ \mu m; \ b = 30 \ \mu m).$ $\underline{B}: \ \ \Delta_1 \ versus \ \lambda_c \ (\Delta_2 = -0.04\%; \ a = 4 \ \mu m; \ b = 30 \ \mu m).$

 $\frac{\omega}{20} = \frac{\omega}{20} + \frac{\omega}{20} \frac{\omega}{20}$



Fig. 13. The dispersion characteristics of basic fiber design ($\alpha = 50$).

structure with a Ge doped cladding tends to form a secondary wavequide and to rise the so-called "leaky modes". For the depressed-cladding structure with $\Delta_1 = 0.2 - 0.3\%$, the optimum core radius is at about 4 μ m, which corresponds to spot size ÷ 5 μ m. At higher wavelengths, field intensity decreases and field edge penetrates into the cladding.

To minimize attenuation it is advisable to keep Ge content in the core low; this in its turn increases sensitivity for micro/macro bending losses. Minimum changes of structural parameters in the core Δ_1 , a) cause a significant shift of λ_c and D_0 ; but their influence on the dependence of w_{∞}/\bar{w} is relatively small. Structures characterised by high index differences in the cladding are very sensitive to micro/macro bending losses; for this reason such fibers are suited for sensing pressure changes.

From our own results it follows that it is very important to have a technology enabling reliable reproduction of fiber parameters a and Δ_1 , because these parameters influence transmission parameters more significantly than the central dip. For producing more complicated structures it is therefore desirable to use more precise technologies, e.g. PCVD.

Computer modelling allows to determine the transmision characteristics resulting from various fiber design modifications (index profile, core geometry, dopant changes etc.); in this manner it is possible to propose structures for sensing devices (sensitive to pressure, temperature, electromagnetic field etc.). It would be possible to include the influence of these external factors into the formula for chromatic dispersion, which is influenced by material dispersion and propagation constant.

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PŘENOSOVÉ CHARAKTERISTIKY JEDNOVIDOVÝCH VLÁKEN

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Článek popisuje užití matematických algoritmů pro výpočet přenosových charakteristik libovolných kruhově symetrických jednovidových struktur při zvolené vlnové délce. Pro přenos se volí vlnová délka vyšší než cutoff vlnová délka. Sledovány jsou tyto přenosové charakteristiky: cut-off vlnová délka, chromatická disperze a vzájemný poměr vazebných a ohybových ztrát. Cílem bylo určit vliv designu (geometrické struktury) vlákna na sledované přenosové charakteristiky. Vhodné parametry designu vlákna jsou diskutovány v článku. Závislost indexu lomu na vlnové délce, která je charakteristická pro dané složení jádra i obalu vlákna, je zahrnuta pomocí Sellmeierových koeficientů.

- Obr. 1. Schematické zobrazení struktur SM vláken.
 - $\Delta_1 \text{ znamená: } \Delta_1 = (n_1 n_c)n_c 100;$
 - Δ_2 znamená: $\Delta_2 = (n_2 n_c)n_c 100;$

nc označuje čistý SiO₂.

- Obr. 2. Základní strukturní typy SM vláken:
 - A) snížený obal $\Delta_1 = 0.30\%$, $\Delta_2 = -0.04\%$;
 - B) vyrovnaný obal $\Delta_1 = 0.34\%$;
 - C) obal dopovaný Ge $\Delta_1 = 0.34\%$, $\Delta_2 = 0.06\%$;
 - D) plně fluorem dopovaná struktura (FFD)
 - $\Delta_1 = 0.00\%, \ \Delta_2 = -0.34\%.$
- Obr. 3. Schematické zobrazení centrálního dipu (Rovnice (7) platí pro $0 \le R < 1$ a rovice (8) pro $0 \le R \le w_d/a$. A: Popis rovnicí (7);
 - B: Popis rovnicí (8).
- Obr. 4. Vliv měnícího se indexu lomu jádra (Δ_1 na nulovou chromatickou disperzi při vlnové délce 1300 nm. Parametry vláken:
 - + $\Delta_1 = 0.4\%$; $\Delta_2 = -0.04\%$; $b = 30 \, \mu m$;
 - $\Delta_1 = 0.3\%; \ \Delta_2 = -0.04\%; \ b = 30 \mu m;$
 - ▲ $\Delta_1 = 0.2\%$; $\Delta_2 = -0.04\%$; $b = 30 \mu m$.
- Obr. 5. Vliv změny indexu lomu jádra (Δ_1) na w_{∞}/\bar{w} . Parametry vláken:
 - + $\Delta_1 = 0.4\%$; $\Delta_2 = -0.04\%$; $b = 30 \,\mu m_i$
 - $\Delta_1 = 0.3\%$; $\Delta_2 = -0.04\%$; $b = 30 \mu m$;
 - ▲ $\Delta_1 = 0.2\%$; $\Delta_2 = -0.04\%$; $b = 30 \mu m$.
- Obr. 6. Vliv šířky dipu aproximovaného přímkou na D_0 při počáteční hodnotě $D_0 = 0.0$ ps/km.nm ($\lambda = 1300$ nm); $\Delta_1 = 0.384\%$, $\Delta_2 = -0.075\%$, $a = 3.96 \,\mu$ m, $b = 33.3 \,\mu$ m.
- Obr. 7. Závislost D_0 při změně poloměru <u>a</u> a rozdílné hodnoty Δ_2 . Parametry vláken:
 - $\Delta_1 = 0.20\%$; $\Delta_2 = -0.04\%$; $b = 30 \,\mu m$;
 - + $\Delta_1 = 0.20\%$; $\Delta_2 = -0.14\%$; $b = 30 \mu m$.

Obr. 8. Hodnota vztahu w_{∞}/\bar{w} jako funkce hloubky sníženého obalu. (Uspokojiví hodnota w_{∞}/\bar{w} je očekávána pro strukturu s $\Delta_1 = 0.3\%$ při poloměru <u>a</u> $\in <$ 4.4; 5.0 µm >). Parametry vláken: ▲ $\Delta_1 = 0.2\%$; $\Delta_2 = -0.14\%$; $b = 30 \, \mu m$; + $\Delta_1 = 0.3\%$; $\Delta_2 = -0.04\%$; $b = 30\mu m$; • $\Delta_1 = 0.3\%$; $\Delta_2 = -0.03\%$; $b = 30 \, \mu m$; $\Delta_1 = 0.3\%; \ \Delta_2 = -0.02\%; \ b = 30 \mu m.$ Obr. 9. Vliv b/a na průběh w_{∞}/\bar{w} . Parametry vláken: + $\Delta_1 = 0.15\%$; $\Delta_2 = -0.04\%$; $a = 4 \, \mu m$; • $\Delta_1 = 0.20\%; \ \Delta_2 = -0.04\%; \ a = 4\mu m;$ • $\Delta_1 = 0.30\%; \ \Delta_2 = -0.04\%; \ a = 4 \ \mu m;$ • $\Delta_1 = 0.40\%$; $\Delta_2 = -0.04\%$; $a = 4 \, \mu m$. Obr. 10. Závislost hodnot w_{∞}/\bar{w} pro základní strukturní typy. Obr. 11. Vliv relativní hloubky dipu y na cut-off vlnovou délku λ_c . Parametry vlákna: $\Delta_1 = 0.384\%$; $\Delta_2 =$ -0.075%; $a = 2.5 \ \mu m$; $b = 33.3 \ \mu m$. Relativní šířka *dipu: I*: $\rho = 50$; *II*: $\rho = 5$; *III*: $\rho = 1$. Obr. 12. Vliv změny α mocniného profilu jádra a Δ_1 na pozici cut-off vlnové délky. A: α profil versus λc ($\Delta_1 = 0.30\%$; $\Delta_2 = -0.04\%; \ a = 4 \ \mu m; \ b = 30 \ \mu m).$ B: Δ_1 versus λ_c ($\Delta_2 = -0.04\%$; $a = 4 \mu m$; $b = 30 \ \mu m; \ \alpha = 40$).

Obr. 13. Disperzní charakteristiky základních struktur vláken ($\alpha = 50$).