BENDING OF PIEZOELECTRIC ACTUATORS

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An analytical solution of total deformations and stresses due to piezoelectric deformations and to external force is given for piezoelectric bending actuators composed generally of n layers. The deflection w(L) of the free end and the blocking force P_B are discussed in detail for actuators composed of two or three layers. Actuators made from piezoelectric ceramics show some deviations, i.e. hysteresis and non-linearity, from predicted linear dependence of w(L) on the applied voltage. These deviations are due to the voltage dependence of piezoelectric parameters in this material.

INTRODUCTION

The piezoelectric effect leading to elongation or contraction under electric field is used in different types of actuators [1]. If the displacements larger than ca 100 μm and not too high forces are required, bimorph bending elements are applied [2-4].

The basic function of bimorphs is shown in figure 1. Two plates of piezoelectric ceramics with the orientation of the remanent polarization (after poling by high dc electric field at elevated temperature) represented by arrows are firmly joined by gluing or soldering. If the electric field E_z acted in the direction of remanent polarization on one free plate of length L, the plate would contract by $\Delta L = d_{31} E_z L$ where the piezoelectric constant $d_{31} \approx -200 \times 10^{-12}$ up to -300×10^{-12} m V^{-1} for the usually used PZT type ceramics [5]. For the opposite orientation of the electric field and remanent polarization, the plate would elongate. Because of joining of two plates, the bimorph will bend up (as in figure 1) or down, depending on the used polarity.

Figure 1*a* shows the joining of plates with an antiparallel orientation of the remanent polarization and with electrodes on the upper and lower surface of the bimorph. In the bimorph shown in figure 1b with a parallel orientation of remanent polarizations in both plates, the third central electrode is added so that half the voltage is sufficient for the same bending. In bimorphs in figure 1*a* and 1*b*, the voltage on one of the plates has the opposite polarity than that used originally for poling. The electric field used for excitation of bimorph is relatively high, corresponding to the voltage difference of 100 up to 200 V for plates 0.2 to 0.5 mm thick. A gradual depolarization of one plate and a decrease of the piezoelectric activity may take place. Therefore, the arrangement shown in figure 1*c* is often used: when the plate *2* is excited by the electric field parallel to the original poling field while plate *1* is short- circuited the bimorph bends up. For opposite-side bending is the plate *1* excited and the plate *2* short-circuited. The bending of the bimorph is then smaller than in figure 1b, however, no depolarization takes place.

The actuators are realized as laminated composite plates. The bimorph in figure 1*a* contains, besides the two piezoelectric plates, also two thin layers of electrodes and one thin central layer of glue or solder. The bimorphs in figures 1*b* or 1*c* contain moreover the central electrode (realized as a metal foil or a prepreg with graphite fibres) with thin layers of glue. The additional layers, especially the central electrode, can considerably influence the bending of the actuator.

The bending of the bimorph composed of two layers has been analysed e.g. in [6]. The case of a thick laminated plate covered by two thin piezoelectric layers has been studied in [7].

The theory of bending of actuators should give the values of two practical quantities: the bending $w(L)$ at the free end of the actuator and the blocking force P_B, i.e. the force at the free end which eliminates the bending.

In this paper, the elastic solution of bending of a laminated plate composed generally of n layers (fixed at one end, i.e. of a laminated cantilever beam) due to piezoelectric deformations and to a force exerted at the free end will first be summarized. The cases of the plate composed of two or three layers will then be treated in more detail and the choice of the parameters leading to maximum bending $w(L)$ or to maximum blocking force P_B will be discussed. Finally, theoretical predictions will be compared with experimental results.

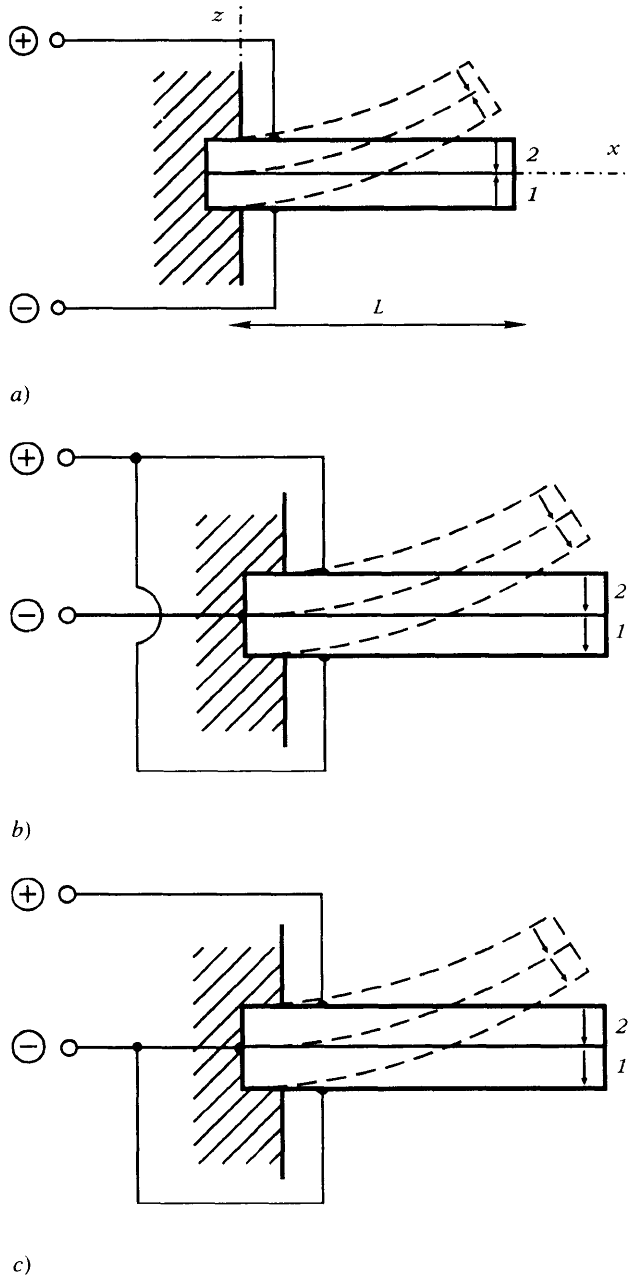

Figure 1. Three cases of excitation of bending actuators. The arrows show the directions of remanent polarization.

BENDING OF ACTUATORS COMPOSED OF n LAYERS

Formulation of the problem

The layers of thickness $h_i = z_i - z_{i-1}$, $i = 1,2, ..., n$ are parallel to the xy plane (see figure 2a). The electric field $E_{zi} = (U_i - U_{i-1}) / h_i$ (where U_i is the voltage applied at z_i) imposes in the i-layer the inelastic deformation (i.e. the piezoelectric deformation which should appear in a free element)

$$\varepsilon_{xxi} = \varepsilon_{yyi} = \varepsilon_i = \mp |E_{zi} d_{31i}| ,$$

$$\varepsilon_{zzi} = \pm |E_{zi} d_{33i}| , \qquad (1)$$

$$\varepsilon_{xyi} = \varepsilon_{xzi} = \varepsilon_{yzi} = 0 .$$

The upper and lower signs are valid if the electric field and the spontaneous polarization are parallel or antiparallel, respectively.

Because of the elastic interaction with other layers, elastic deformation e_{mni} (inducing stresses σ_{mni}) will be added to the piezoelectric deformation ε_{mni} in the i-layer so that the total deformations $e^T_{mni} = \varepsilon_{mni} + e_{mni}$ will build up the bending (complemented possibly by elongation or contraction) of the composite plate.

The linear theory of elasticity can be applied as $|\varepsilon_i| << 1$, $|\varepsilon_{zzi}| << 1$. The actuators are constructed as thin beams of thickness

$$H = \sum_{i=1}^{n} h_i << L$$

so that the theory for thin beams can be used (see e.g. [8]). The edge effects at the clamped and free ends will be neglected.

The layers will be considered elastically isotropic, characterized by Young's modulus E_i and Poisson's ratio ν_i. In fact, the PZT poled ceramics are slightly anisotropic, however, with the so-called transverse isotropy [9] so that only the isotropic constants E_i and ν_i in the xy plane will influence the bending.

There is a free dilatation in the z direction perpendicular to the layers so that the component ε_{zzi} of the piezoelectric deformation does not influence the bending and the stress component $\sigma_{zzi} = 0$. Because of the problem symmetry, the non-diagonal components of the stress and deformation tensors will be zero and there are only two non-zero stress components, σ_{xxi} and σ_{yyi}. They are connected with the elastic deformations by Hooke's law,

$$e_{xxi} = e^T_{xxi} - \varepsilon_i = (1/E_i)(\sigma_{xxi} - \nu_i \sigma_{yyi}) ,$$

$$e_{yyi} = e^T_{yyi} - \varepsilon_i = (1/E_i)(\sigma_{yyi} - \nu_i \sigma_{xxi}) . \qquad (2)$$

Three modes of the plate deformation can be distinguished.

1. Plane stress

For the actuator thin in the y direction, i.e. with width $W < h_i$, there is a free dilatation also in the y direction so that the stress component $\sigma_{yyi} = 0$. The only non-zero component of stress is $\sigma_{xxi} = \sigma_i$ and the important strain components will be $e^T_{xxi} = e^T_i$, $e_{xxi} = e_i$ and ε_i. The total deformation e^T_i must correspond to ben-

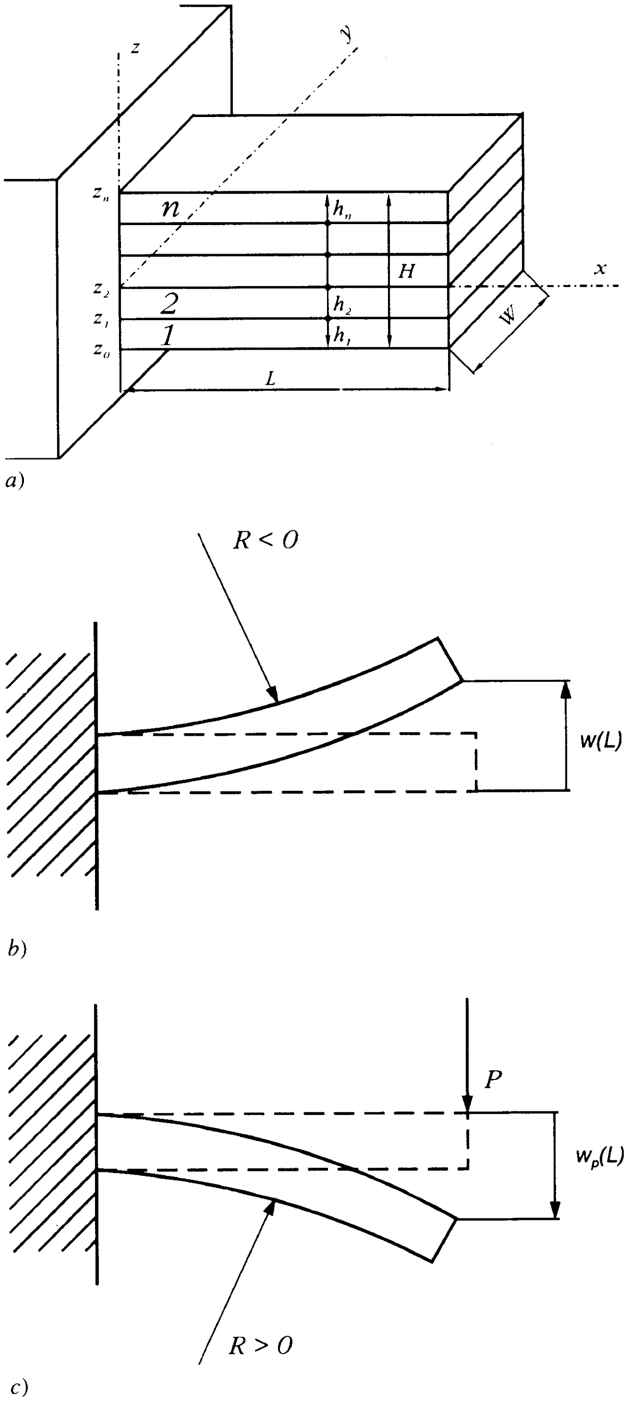

Figure 2.
a) Actuator composed of *n* layers, *b)* bending due to piezoelectric deformations ε_i, *c)* bending due to external force *P*.

ding and must be a linear function of z in the whole laminated plate (see e.g. [8]),

$$e_i^T = \varepsilon_i + e_i = Az + B, \qquad z_0 \le z \le z_n \ , \qquad (3)$$

with so far unknown constants *A*, *B*. The stress component σ_i then follows from Hooke's law (2) as

$$\sigma_i = E_i (Az + B - \varepsilon_i) \ , \qquad z_{i-1} < z < z_i \ . \qquad (4)$$

2. Plane strain

For the actuator thick in the y direction, $W >> h_i$, firmly clamped at $x = 0$, the total deformation $e_{yyi}^T = 0$ so that the elastic deformation $e_{yyi} = -\varepsilon_i$. If we denote again $e_{xxi}^T = e_i^T$, $e_{xxi} = e_i$, $\sigma_{xxi} = \sigma_i$, the total deformation is

$$e_i^T = (1 + \nu_i)\ \varepsilon_i + e_i = Az + B\ , \quad z_0 \le z \le z_n \qquad (3')$$

and Hooke's law gives

$$\sigma_i = [E_i / (1 - \nu_i^2)]\ [Az + B - (1 + \nu_i)\ \varepsilon_i], \qquad z_{i-1} < z < z_i \ . \qquad (4')$$

The second stress component, $\sigma_{yyi} = \nu_i\ \sigma_i - E_i\ \varepsilon_i$, is not important for the discussion of bending.

Equations (3'), (4') can be changed to the form of (3), (4) if

$$E_i' = E_i / (1 - \nu_i^2), \qquad \varepsilon_i' = (1 + \nu_i)\ \varepsilon_i \quad (5')$$

are written instead of E_i, ε_i in equations (3), (4).

The cases 1 and 2 correspond to the so-called cylindrical bending.

3. Plate state

For the actuator thick in the *y* direction, with not too firm clamping at $x = 0$ and at larger distances from the fixed end, the „spherical bending" can be expected: the plate can bend not only along the *x* direction, but also along the *y* direction. In this case, the deformations and stresses are isotropic in the *xy* plane, $e_{xxi}^T = e_{yyi}^T = e_i^T$, $e_{xxi} = e_{yyi} = e_i$, $\sigma_{xxi} = \sigma_{yyi} = \sigma_i$ so that the total deformations will be

$$e_i^T = \varepsilon_i + e_i = Az + B, \qquad z_0 \le z \le z_n \qquad (3'')$$

and the stresses from Hooke's law follow as

$$\sigma_i = [E_i/(1 - \nu_i)]\ [Az + B - \varepsilon_i], \quad z_{i-1} < z < z_i \ . \qquad (4'')$$

This state can again be described by equations (3), (4) if, instead of E_i, changed elastic constants E_i'' are written,

$$E_i'' = E_i/(1 - \nu_i) \qquad (5'')$$

this time with unchanged values ε_i.

In the following treatment, equations (3), (4) will be used with Young's modulus denoted as E_i and the piezoelectric deformations as ε_i. For larger width *W* of the actuator, the values corresponding to (5') or (5'') should be used.

The constants A, B can be calculated from the equations of equilibrium of forces and moments,

$$\int_{z_0}^{z_n} \sigma(z)\,dz = 0, \quad \int_{z_0}^{z_n} z\,\sigma(z)\,dz = M_{ext} \quad , \qquad (6)$$

where $\sigma(z)$ is given by equations (4) and the moment M_{ext} (on the unit length in the y direction) from the external force P is equal to

$$M_{ext} = (P/W)\,(L - x)\ . \qquad (7)$$

According to figure 2c, force P is taken positive if it acts downwards, in the $-z$ direction.

Within the linear theory of elasticity, the solution can be given in three steps:

1. the effect of the piezoelectric deformation ε_i only, with $M_{ext} = 0$. In this case, A and B will be constants.
2. the effect of force P, i.e. of the external moment M_{ext} only, with $\varepsilon_i = 0$. In this case, A and B will be proportional to $(L - x)$ and will be denoted as A_p, B_p.
3. the combined effect of ε_i and M_{ext} can be taken as a sum of cases 1 and 2.

Effect of piezoelectric deformation ε_i

After inserting stresses (4) into equations of equilibrium (6) with $M_{ext} = 0$, two algebraic linear equations for A and B follow,

$$SA + FB = N\ , \qquad (8)$$

$$IA + SB = M$$

where

$$F = \sum_{i=1}^{n} E_i h_i\,,\ S = (1/2)\sum_{i=1}^{n} E_i\,(z_i^2 - z_{i-1}^2)\ ,$$

$$I = (1/3)\sum_{i=1}^{n} E_i\,(z_i^3 - z_{i-1}^3),\ N = \sum_{i=1}^{n} \varepsilon_i E_i h_i\ ,$$

$$M = (1/2)\sum_{i=1}^{n} \varepsilon_i E_i\,(z_i^2 - z_{i-1}^2)\ ,$$

$$h_i = z_i - z_{i-1}\ . \qquad (9)$$

Constants A and B are given by expressions

$$A = (FM - SN)\ /\ (FI - S^2)\ ,$$

$$B = (IN - SM)\ /\ (FI - S^2) \qquad (10)$$

where $F > 0$, $S > 0$, $I > 0$ and also $(FI - S^2) > 0$. The stresses $\sigma_i(z)$ follow from equations (4).

These results are in agreement with the solution of similar problems where inelastic deformations ε_i are of different origin, e.g. due to thermal expansion in [10, 11].

The plate will bend in the xz plane with the radius of curvature $R\ (z) = [1 + e^T\ (z)]\ /\ A$. However, $|e^T(z)| << 1$ and, for a thin plate, $L >> H$, radius of curvature can be taken independent of z and x,

$$R = 1/A \qquad (11)$$

and can be measured at the upper or lower surface of the plate. The constant A has the meaning of the plate curvature. The sign of R was chosen so that for $R < 0$ the center of curvature is in the upper half space (as in figure 2b) and for $R > 0$ it is in the lower half space (as in figure 2c).

The plate clamped at $x = 0$ will bend at the free end $x = L$ by the displacement $w(L)$ which can be calculated from the relation

$$R^2 = L^2 + (R + w(L))^2\ .$$

For $|R| > L >> w(L)$, it is $w(L) \approx -(1/2)\ L^2/R$,

or

$$w(L) \approx -(1/2)\,L^2 A = (1/2)\,L^2\,(SN - FM)\,/\,(FI - S^2)\ . \qquad (12)$$

Effect of external force P

In the case of a laminated cantilever beam under force P acting in the $-z$ direction at the end $x = L$, i.e. under moment M_{ext} given by equation (7) and without inelastic deformations, $\varepsilon_i = 0$, the total deformations e^T_i are equal to the elastic deformations e_i,

$$e_i^T = e_i = A_p z + B_p\ , \qquad z_0 \le z \le z_n \qquad (13)$$

and the stresses follow as

$$\sigma_i = E_i\,(A_p z + B_p)\ , \qquad z_{i-1} < z < z_i\ . \qquad (14)$$

The equations of equilibrium (6) give two linear algebraic equations for A_p and B_p,

$$SA_p + FB_p = 0\ , \qquad (15)$$

$$IA_p + SB_p = (P/W)\ (L - x)$$

with the solution

$$A_p = [F/(FI - S^2)]\ (P/W)\ (L - x)\ , \qquad (16)$$

$$B_p = -[S/(FI - S^2)]\ (P/W)\ (L - x)\ .$$

The constants F, S and I are again given by equations (9). Deformation (13) and stresses (14) are now functions not only of z but also of x. The quantity A_p has again the meaning of curvature, however, this time of the local curvature, $A_p(x) = 1/R(x)$. It can be written approximately in the form $A_p(x) = -d^2w_p(x)\ /\ dx^2$ where $w_p(x)$ is the local bending displacement. The solution of the differential equation $d^2w_p\ (x)/dx^2 = C(L-x)$ where $C = -[F/(FI\ -S^2)]\ (P/W)$ with the boundary conditions $w_p(0) = 0$ and $dw_p(x)/dx = 0$ for $x = 0$ gives $w_p\ (x) = C[(Lx^2/2) - x^3/6]$. Therefore, the bending $w_p(L)$ at the beam end is $w_p(L) = (1/3)\ C\ L^3$, i.e.

$$w_p(L) = -(1/3)L^3\ [F/(FI - S^2)]\ (P/W) = -(1/3)L^2 A_p(0)\ . \quad (17)$$

For $P > 0$ (i.e. in the direction $-z$, see figure 2*c*) it is $R(x) > 0$, $A_p(x) > 0$ and $w_p(L) < 0$.

Combined effect of ε_i and P

For a simultaneous action of piezoelectric deformations ε_i and external force P, the resulting stresses are given by a sum of equations (4) and (14), the resulting total deformations by a sum of (3) and (13) and the resulting bending at the end $w_r(L)$ by a sum of (12) and (17), i.e.

$$w_r(L) = w(L) + w_p(L) = -L^2\ [(1/2)A + (1/3)A_p(0)] = $$
$$= [L^2/(FI - S^2)]\ [(1/2)\ (SN - FM) - (1/3)FLP/W] \quad (18)$$

where the constants L, W, F, S, I and $(FI - S^2)$ are positive.

For a clear discussion it will be assumed that the actuator is excited so that bending $w(L)$ due to ε_i is directed upwards, i.e. $w(L) > 0$. Then $A < 0$ and the first term in the brackets in equation (18), $(SN - FM) > 0$. The force $P > 0$ bends the beam downwards, $w_p(L) < 0$ and the second term in the bracket remains negative.

If the beam end meets an obstacle at a given distance $w_r(L)$, $0 \le w_r(L) < w(L)$, the force P which the obstacle will exert on the beam end can be calculated from (18) as

$$P = (3W/LF)[(1/2)(SN - FM) - (1/L^2)\ (FI - S^2)\ w_r(L)]\ . \quad (19)$$

The blocking force P_B corresponds to $w_r(L) = 0$, when the piezoelectric bending $w(L)$ is eliminated by bending $w_p(L)$, i.e. when $w_p(L) = -w(L)$,

$$P_B = (3/2)\ (W/L)\ (SN - FM)\ /\ F\ . \quad (20)$$

The general expression (19) for force P can be rewritten, using (20), in the form

$$P = P_B\ [1 - w_r(L)\ /\ w(L)]\ . \quad (21)$$

ACTUATOR COMPOSED OF TWO LAYERS

General case

A general case will first be considered (figure 3): plate 1 and plate 2 are characterized by constants h_1, E_1, ν_1, ε_1 and by h_2, E_2, ν_2, ε_2, respectively, and common dimensions L, W. The origin of coordinate z will be chosen in the interface $z_1 = 0$ so that $z_0 = -h_1$ and $z_2 = h_2$. The results can be directly written as a special case of equations (1) – (21) for $n = 2$.

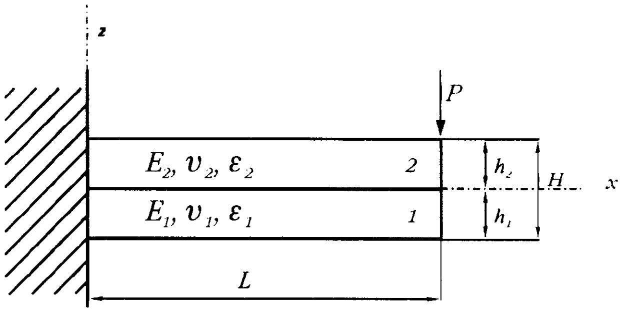

Figure 3. Actuator composed of two plates 1, 2.

Only the expressions for bendings $w(L)$ and $w_p(L)$ and for blocking force P_B will be given in detail, using the dimensionless parameters

$$k = h_1/h_2\ ,\quad K = E_1/E_2\ . \quad (22)$$

Bending $w(L)$ due to piezoelectric deformations ε_1, ε_2 follows from equations (12) and (9) as

$$w(L) = 3(\varepsilon_1 - \varepsilon_2)\ (L^2/h_2)\ f(k, K) \quad (23)$$

where the dimensionless function $f(k, K) > 0$,

$$f(k, K) = \frac{k(1 + k)K}{1 + 2k(2k^2 + 3k + 2)K + k^4 K^2}\ , \quad (24)$$

in accordance with the results obtained by another method e.g. in [6, 11].
Bending $w_p(L)$ due to force P is given by (17) and (9) as

$$w_p(L) = -4(L^3/h_2^3)\ (1/E_2)\ (P/W)\ g(k, K) \quad (25)$$

where $g(k, K) > 0$,

$$g(k, K) = \frac{1 + kK}{1 + 2k(2k^2 + 3k + 2)K + k^4 K^2}\ . \quad (26)$$

The blocking force for $w_r(L) = 0$, i.e. for $w_p(L) = -w(L)$, follows from (20) and (9) (or (23) and (25)) as

$$P_B = (3/4)\ E_2\ (h_2^2/L)\ W(\varepsilon_1 - \varepsilon_2)\ p(k, K) \quad (27)$$

where $p(k, K) > 0$,

$$p(k, K) = f(k, K)\ /\ g(k, K) = k(1 + K)\ K\ /\ (1 + kK)\ . \quad (28)$$

For another chosen distance $w_r(L) \neq 0$, the force P is given by equation (21).

A possible maximization of the piezoelectric bending $|w(L)|$ and of the blocking force P_B will now be discussed. For the chosen values of $(\varepsilon_1 - \varepsilon_2)$, L, h_2, the bending $w(L)$ from (23) is proportional to $(\varepsilon_1 - \varepsilon_2)$ (L^2/h_2) and to the dimensionless function $f(k, K)$: the values $k = h_1\ /\ h_2$ and $K = E_1\ /\ E_2$ should be found for maximum of $f\ (k, K)$. However, this function of two variables has no local maximum for finite values of k, K.

For the chosen value K_0 of K, the conditional maximum of $f(k, K_0)$ follows from the condition $\partial f(k, K_0)/\partial k = 0$ for $k = k_M$, i.e. from the relation $k_M^2(3 + 2k_M) = 1/K_0$.

For the chosen value k_0 of k, the conditional maximum of $f(k_0, K)$ follows from $\partial f(k_0, K)/\partial K = 0$ for $K = K_M$, i.e. from the relation $K_M = 1/k_0^2$.

However, these two conditions have no common solution for finite values of k and K. In both cases the function $f(k, K)$ approaches the maximum $f(k, K) = 1/4$ for $K_0 \to \infty$, $k_M \to 0$ or $k_0 \to 0$, $K_M \to \infty$.

In practical cases, high values of K and small values of k should be chosen for higher values of $f(k, K)$. For example, for plate 2 of piezoelectric PZT with $E_2 = 70$ GPa, plate 1 with higher E_1 should be chosen e.g. of steel with $E_1 = 210$ GPa so that $K_0 = 3$. The conditional maximum is then realized for $k_M = 0.304$ leading to the value $f(k_M, K_0) = 0.177$.

The blocking force P_B from (27) is proportional to $E_2(h_2^2/L)\,W\,(\varepsilon_1 - \varepsilon_2)$ and to the dimensionless function $p(k, K)$. This function again has no local maximum for finite values of k, K and is an increasing function of both k and K. For example for $K = 3$, $p(k,3) = 3k\,(1+k)/(1+3k)$ increases with k.

Therefore, for a given value of K, different values of k should be chosen for maximization of $w(L)$ or of P_B.

Special case $E_1 = E_2 = E$ and $h_1 = h_2 = h$

If both plates are of the same PZT material,

$$E_1 = E_2 = E, \qquad K = 1 \tag{29}$$

the functions f, g, p simplify to

$$f(k, 1) = k/(1 + k)^3 \tag{24a}$$

with maximum value for $k = 1/2$ equal to $f(1/2, 1) = 4/27 = 0.14185$,

$$g(k, 1) = 1/(1 + k)^3 \tag{26a}$$

with maximum for $k \to 0$, $g(0, 1) = 1$ and decreasing with increasing k,

$$p(k, 1) = k \tag{28a}$$

increasing with k.

For a bimorph composed of two PZT plates of the same thickness,

$$h_1 = h_2 = h, \qquad k = 1\ , \tag{30}$$

it is $f(1, 1) = 1/8 = 0.125$, $g(1, 1) = 1/8$, $p(1, 1) = 1$.

The choice $k = 1/2$ would lead to a slightly larger bending $w(L)$, however to half blocking force P_B than in the case $k = 1$.

In summary, for the usually used bimorphs with $E_1 = E_2 = E$ and $h_1 = h_2 = h = H/2$, it follows for piezoelectric bending (independent of E)

$$w(L) = (3/8)\,(L^2/h)\,(\varepsilon_1 - \varepsilon_2)\ , \tag{23a}$$

for bending due to force P

$$w_p(L) = -(1/2)\,(L^3/h^3)\,(1/E)\,(P/W) \tag{25a}$$

and for the blocking force

$$P_B = (3/4)\,EW\,(h^2/L)\,(\varepsilon_1 - \varepsilon_2)\ . \tag{27a}$$

If the absolute value of the piezoelectric deformation is denoted as $\varepsilon = |E_z\, d_{31}|$ and when both plates are excited, $\varepsilon_1 = \varepsilon$ (elongation) and $\varepsilon_2 = -\varepsilon$ (contraction) for $w(L) > 0$ and $(\varepsilon_1 - \varepsilon_2) = 2\varepsilon$. For the short-circuited layer *1* , $\varepsilon_1 = 0$, $\varepsilon_2 = -\varepsilon$ and $(\varepsilon_1 - \varepsilon_2) = \varepsilon$. Therefore, the values of $w(L)$ and P_B in the first case are twice larger than in the second case.

ACTUATOR COMPOSED OF THREE LAYERS

The usual arrangement will be assumed (figure 4): plate *1* and *3* are of the same piezoelectric material and of the same thickness while central plate *2* is of a non-piezoelectric material with generally another thickness. The actuator will be characterised by the constants

$$E_1 = E,\ \nu_1 = \nu,\ h_1 = h,\ \varepsilon_1\ ,$$

$$E_2 = E_0,\ \nu_2 = \nu_0,\ h_2 = 2h_0,\ \varepsilon_2 = 0\ ,$$

$$E_3 = E,\ \nu_3 = \nu,\ h_3 = h,\ \varepsilon_3\ . \tag{31}$$

and by common dimensions L, W; the total thickness is $H = 2(h + h_0)$. The origin of the z - coordinate will be chosen in the middle of the central plate so that $z_0 = -(h + h_0)$, $z_1 = -h_0$, $z_2 = h_0$, $z_3 = (h + h_0)$.

The results then follow from equations (1)–(21) for $n = 3$.

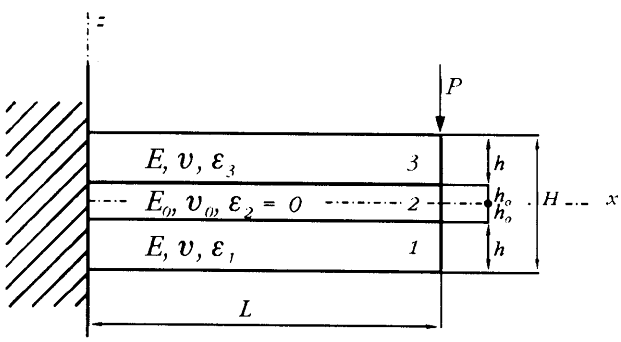

Figure 4. Actuator composed of two piezoelectric plates 1, 3 and of the central plate 2.

The geometrical symmetry with respect to the central plane $z = 0$ simplifies expressions (9) to

$$F = 2(Eh + E_0 h_0), \qquad S = 0 \;,$$

$$I = (2/3)\ \{E[(h + h_0)^3 - h_0^3] + E_0 h_0^3\}\;,$$

$$N = (\varepsilon_1 + \varepsilon_3)\ Eh\;,$$

$$M = -(E/2)\ (\varepsilon_1 - \varepsilon_3)\ [(h + h_0)^2 - h_0^2]\;. \qquad (9b)$$

The zero value of S simplifies equations (10) and (16) to

$$A = M/I\;, \qquad B = N/F\;, \qquad (10b)$$

$$A_p = (1/I)\ (P/W)\ (L - x)\;, \qquad B_p = 0\;. \qquad (16b)$$

Only the expressions for $w(L)$, $w_P(L)$ and P_B will be given, using the dimensionless parameters

$$c = h_0/h\;, \qquad C = E_0/E\;. \qquad (32)$$

Bending $w(L)$ due to piezoelectric deformations ε_1 and ε_3 follows from (12) with (9*b*), (10*b*) as

$$w(L) = (3/8)\ (L^2/h)\ (\varepsilon_1 - \varepsilon_3)\ m(c, C) \qquad (33)$$

where

$$m(c, C) = \frac{1 + 2c}{1 + 3c + 3c^2 + c^3 C}\;. \qquad (34)$$

Function $m(c, C)$ decreases with increasing c and C and has its maximum for $c \to 0$ when $m \to 1$ and equation (33) transforms into (23*a*) corresponding to the case without the central plate.

Bending $w_P(L)$ due to force P equals, according to (17) with (9b) and (16b), to

$$w_P(L) = -\ (1/2)\ (L^3/h^3)\ (1/E)\ (P/W)\ n(c, C) \qquad (35)$$

where

$$n(c, C) = \frac{1}{1 + 3c + 3c^2 + c^3 C}\;. \qquad (36)$$

The function $n(c, C)$ has maximum value $n = 1$ for $c \to 0$ when (35) transforms to (25*a*).

The blocking force P_B follows from condition $w_P(L) = -w(L)$ (or from (20) with (9*b*) and (16*b*)) as

$$P_B = (3/4)\ EW\ (h^2/L)\ (\varepsilon_1 - \varepsilon_3)\ q(c) \qquad (37)$$

where

$$q(c)\ m(c, C)\ /\ n(c, C) = 1 + 2c = 1 + 2h_0/h\;. \qquad (38)$$

P_B does not depend on the elastic constant E_0 of the central plate and increases with its thickness $2h_0$. For $2h_0 \to 0$, equation (37) transforms to (27a). For $w_r(L) \neq 0$, the force P is again given by (21).

If both plates 1 and 3 are excited and $\varepsilon_1 = \varepsilon$, $\varepsilon_3 = -\varepsilon$, it is $(\varepsilon_1 - \varepsilon_3) = 2\varepsilon$ ($w(L) > 0$ for $\varepsilon > 0$). In this case, in equations (9*b*) and (10*b*) $N = 0$ and $B = 0$. If the plate 1 is short-circuited, $\varepsilon_1 = 0$, $\varepsilon_3 = -\varepsilon$ and $(\varepsilon_1 - \varepsilon_3) = \varepsilon$. Again, the values $w(L)$ and P_B in the first case are twice larger than in the latter case.

Finally, the actuators composed of three layers and of two layers (with $E_1 = E_2 = E$ and $h_1 = h_2 = h$) will be compared. In both cases, the piezoelectric bendings $w(L)$ (equations (33) and (23*a*)) are proportional to L^2/h, i.e. they are larger for longer and thinner actuators and do not depend on width W. The blocking forces P_B (equations (37) and (27a)) are proportional to h^2/L and to W, i.e. on contrary they are larger for shorter and thicker actuators with larger widths W.

For the actuator composed of three layers, moreover the bending $w(L)$ from (33) decreases with increasing thickness $2h_0$ and increasing elastic constant E_0 of the central plate, however, the blocking force P_B from (37) increases with increasing thickness $2h_0$ and does not depend on E_0.

COMPARISON WITH EXPERIMENTS

The actuators were prepared from the piezoelectric ceramics of PZT type, PKM-23 European PiezoCeramics, characterized by the values of piezoelectric constant $d_{31} = -230 \times 10^{-12}$ mV^{-1} and of Young's modulus $E = 65$ GPa. These values were determined from the usually used measurements of the resonant frequency of longitudinal vibrations of piezoelectric plates [12].

In the first set of specimens, the piezoelectric plates (with screen printed and burnt in silver electrodes) of length $l = 45$ mm, width $W = 6$ mm and height $h = 0.28$ mm were joint by a thin layer of silver solder of thickness ≈ 0.02 mm (i.e. , $h_0 \approx 0.01$ mm) so that it was possible to contact also the central electrode.

Other two sets of actuators were prepared with the central electrode of prepreg (with carbon fibres) of thickness 0.1 mm ($h_0 = 0.05$ mm) with two different Young's moduli, $E_0 = 50$ GPa and $E_0 = 120$ GPa.

After gluing or soldering the piezoelectric plates of actuators were poled by d.c. voltage 600 V. The orientation of remanent polarization in poled plates was according to figure 1b. The deflection $w(L)$ of the actuators having the effective free length $L = 35$ mm, connected according to figures 1b or 1c, was measured at the free end using optical microscope.

The dependences $w(L)$ on the applied voltage U predicted from the theory will first be summarized. The effect of electrodes on bending of the actuators without prepreg can be neglected and the values $w(L)$ following from equation (23a) for the above mentioned values of h, L, and d_{31}, with $\varepsilon = |d_{31}|\ U/h$ are given in the table 1.

For the actuator with the central prepreg plate, $c = h_0/h = 0.179$ and $C = E_0/E = 0.759$ or 1.846, $m(c, C = 0.83$ (practically independent of C), $w(L)$ following from equation (33) is also given in the table 1.

Table 1. Theoretical dependences (from equations (23*a*) and (33)) of deflection $w(L)$ in mm on the applied voltage U in V.

without prepreg	both plates excited	$w(L) = 0.270 \times U/100$
	plate *1* short-circuited	$w(L) = 0.135 \times U/100$
with prepreg	both plates excited	$w(L) = 0.224 \times U/100$
	plate *1* short-circuited	$w(L) = 0.112 \times U/100$

The predicted linear dependences of $w(L)$ on U are shown in figures 5-8 by dashed lines, together with the measured dependences.

a) If the remanent polarization and applied electric field are parallel, the absolute value of d_{31} increases with U: remanent polarization increases due to improving arrangement of ferroelectric domains.

b) If the electric field and remanent polarization are antiparallel, the absolute value of d_{31} decreases: the remanent polarization decreases due to disordering of ferroelectric domains.

During the repeated two-side bending cycles these two processes alternate in both plates. The effect of depolarization can be best seen for repeated one-side bending at higher applied voltage from figure 8. The electric field and remanent polarization remains antiparallel in plate 1 all the time for this case and gradual depolarization of plate 1 takes place. After a relatively small number of bending cycles the ferroelectric domain structure becomes disordered which leads to a permanent bending $w_0(L)$ of the actuator. Plate 1 is than no more active and the pre-bent actuator behaves as the actuator with a short-circuited plate 1, with only plate 2 active, as shown in figure 8, curve *b*.

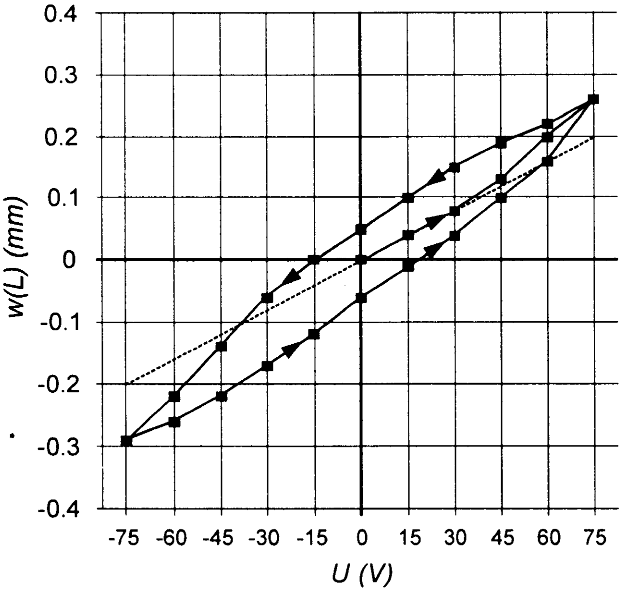

Figure 5. Dependence of $w(L)$ on U for actuator without prepreg, with two plates excited, at low voltages U, two-side bending cycle.

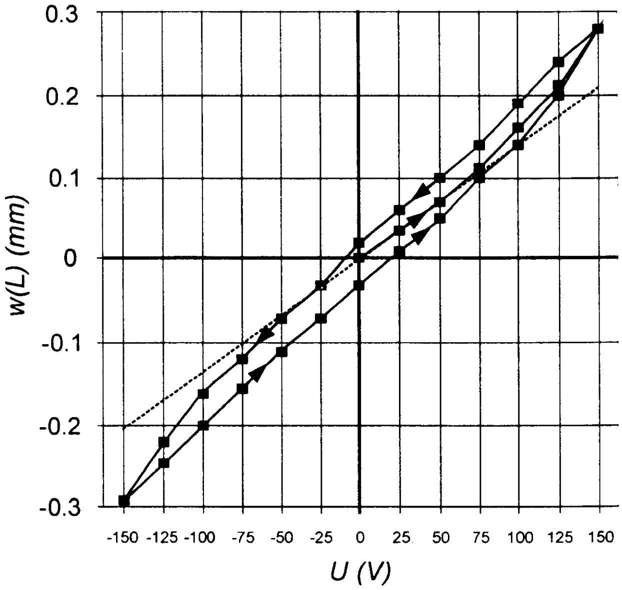

Figure 6. Dependence $w(L)$ on U for actuator without prepreg with one plate short-circuited, two-side bending cycle.

The first cycle of two-side bending in figures 5-7 shows the known non-linearity and hysteresis of the piezoelectric actuators (discussed e.g. in [13-15]) which is due to the dependence of the piezoelectric constant d_{31} on applied voltage U. In the theoretical treatment, the value $d_{31} = -230 \times 10^{-12}$ mV^{-1}, measured at low voltage U, was assumed constant.

The non-linearity and hysteresis is due to two effects:

If plate 1 is short-circuited, the electric field and remanent polarization remain parallel in plate 2 all the time. The deflections of actuator are close to the predicted values with relatively small hysteresis and non-linearity as shown in figure 8, curve *c*.

It is seen from figures 6 and 7 that introduction of the prepreg decreases the deflection $w(L)$ by $\approx$ 17%. However, as it follows from equation (37) and the value $q = 1.375$, it increases the blocking force by $\approx$ 37%.

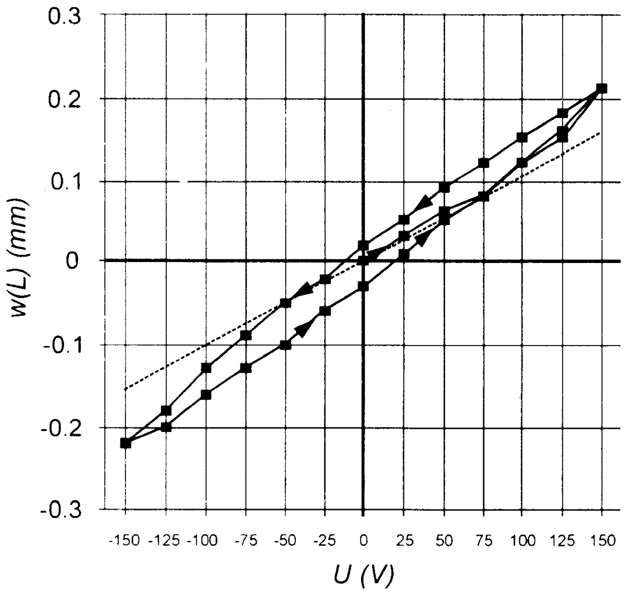

Figure 7. Dependence $w(L)$ on U for actuator with prepreg, with one plate short-circuited, two-side bending cycle.

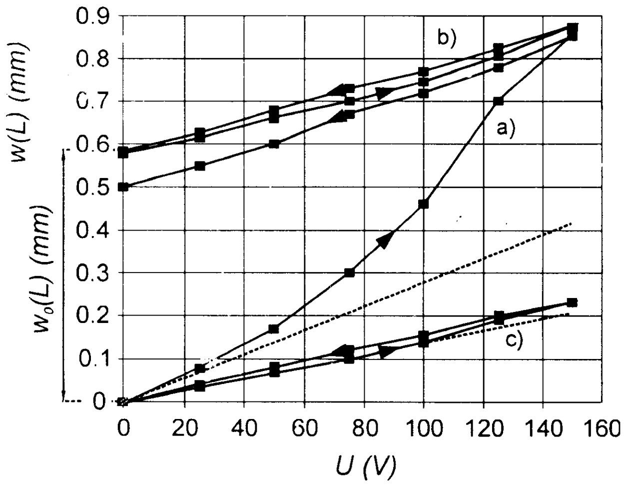

Figure 8. Dependence $w(L)$ on U, actuator without prepreg.
a) two plates excited, first one-side bending cycle
b) two plates excited, after 100 one-side bending cycles
c) plate 1 short-circuited, after 100 one-side bending cycles.

CONCLUSION

The piezoelectric bending actuators can be composed of a higher number of layers and, therefore, the analytical solution of stresses and total deformations due to piezoelectric deformations and external force has been presented for actuators composed generally of n layers. The deflection $w(L)$ and the blocking force P_B have been discussed for actuators composed of two or three layers.

For the special case of two piezoelectric plates of the same thickness h and free length L, deflection $w(L)$ is proportional to L^2/h and to the piezoelectric deformations $\varepsilon = d_{31}\, U/h$ and does not depend on the elastic constants nor on the width W. The blocking force P_B is proportional to h^2/L, ε, W and to Young's modulus E of the piezoelectric material.

An introduction of the third, non-active central plate decreases $w(L)$ and increases P_B.

In the theory, the piezoelectric constant d_{31} has been assumed constant. Within the used linear elastic theory, $w(L)$ and P_B are linearly proportional to the piezoelectric deformations and, therefore, also the applied voltage U.

The measurements of the dependence of $w(L)$ on U have shown deviations from the predicted theoretical values. Experimentally found non-linear behaviour and hysteresis of bending actuators made from piezoelectric ceramics are due to the dependence of the piezoelectric constants on the applied voltage in this material. These effects have to be taken into account especially at higher driving voltages U between 100 and 200 V currently used in applications.

The presented theory describes satisfactorily the behaviour of the bending actuators in the stabilized state, after a higher number of the bending cycles.

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OHYB PIEZOELEKTRICKÝCH MĚNIČŮ

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Piezoelektrické ohybové měniče mohou být složeny z více vrstev: kromě dvou piezoelektrických destiček vyrobených obvykle z piezoelektrické keramiky a vrstev elektrod se často užívá i střední nosná vrstva z jiného materiálu. Je proto podáno analytické řešení celkových deformací a mechanických napětí, způsobených jednak piezoelektrickými deformacemi vlivem přiloženého elektrického pole a jednak vnější silou, obecně pro měnič složený z n vrstev. Podrobně je pak diskutován průhyb $w(L)$ volného konce a blokovací síla P_B pro měniče složené ze dvou a ze tří vrstev.

Ve speciálním případě dvou piezoelektrických destiček stejné tloušťky h a délky L je průhyb $w(L)$ úměrný L^2/h a piezoelektrickým deformacím $\varepsilon = d_{31}\ U/h$, kde d_{31} je piezoelektrická konstanta. Blokovací síla P_B je úměrná h^2/L, ε, šířce měniče W a Youngovu modulu E. Užití třetí střední nosné destičky vede ke snížení průhybu $w(L)$, avšak ke zvýšení blokovací síly P_B.

V teorii se předpokládalo, že piezoelektrická konstanta d_{31} nezávisí na aplikovaném elektrickém napětí U, takže $w(L)$ i P_B by měly být lineárně úměrné U. Měření však ukázala nelineární závislost $w(L)$ na U a hysterezi měničů. Tyto efekty jsou způsobeny závislostí d_{31} na napětí U, která je typická pro piezoelektrickou keramiku a uplatňuje se při prakticky užívaných budících napětích mezi 100 až 200 V.

Po větším počtu ohybových cyklů se však v ustáleném stavu blíží závislost $w(L)$ na U závislosti lineární a vlastnosti ohybových měničů jsou pak uspokojivě popsány předloženou teorií.