EFFECT OF INTERFACE PROPERTIES ON THE NONLINEAR BEHAVIOUR OF
LONG-FIBRE-REINFORCED UNIDIRECTIONAL CERAMIC-MATRIX
MINI-COMPOSITES SUBJECTED TO TENSILE AND FATIGUE LOADING

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Abstract

In this paper, the effect of the interface properties between the fibre and the matrix on the nonlinear behaviour of long-fibre-reinforced unidirectional ceramic-matrix mini-composites (mini-CMCs) is investigated. The tensile nonlinear constitutive relationship and fatigue loading/unloading constitutive relationship are developed considering the different damage mechanisms of cracking in the matrix, debonding in the interface, and gradually the broken fibres. The relationships of the interface properties between the fibre and the matrix, the nonlinear tensile strain, and the fatigue loading/unloading hysteresis loops are established. The effects of the interface properties between the fibre and the matrix on the nonlinear tensile and fatigue loading/unloading damage evolution are analysed. The nonlinear tensile strain at the damage stage and the fracture strain of mini-CMCs decrease with the strong interface properties; and the fatigue loading/unloading hysteresis loops area and strain decrease, and the hysteresis modulus increases with the strong interface properties. The experimental tensile stress-strain curves and fatigue loading/unloading hysteresis loops of the Hi-Nicalon™ and Tyranno™ SiC/SiC mini-composites are predicted corresponding to the different interface properties.

Keywords Ceramic-matrix composites (CMCs); Mini; Matrix cracking; Interface debonding; Fibre failure; Hysteresis loops.
1. Introduction

Ceramic-matrix composites (CMCs) are new types of thermal-structural-functional integrated materials with the advantages of metal materials, ceramic materials and carbon materials [Chyba! Nenalezen zdroj odkazů.]. They have the characteristics of material-structural integration. Through the optimisation design of each structural unit, synergistic effects can be produced, and the high performance and reasonable matching of each performance can be achieved. Therefore, CMCs have high temperature resistance, corrosion resistance, wear resistance, a low density, a high specific strength and modulus, a low thermal expansion coefficient, insensitivity to cracks, no catastrophic damage and other advantages [Chyba! Nenalezen zdroj odkazů.]. Since the 1990s, applications of CMCs have already been carried out in Europe and the United States on a demonstration and verification platform with thrust-to-weight ratios of 8-10 aeroengines (i.e., F119, EJ200, F414, M88-III, TRENT 800, etc.). The results show that fibre-reinforced CMCs can reduce the weight of stationary parts under moderate load by more than 50%, and significantly improve their fatigue life. Generally speaking, intermediate-temperature and medium-load stationary parts such as a nozzle regulator/seal have already completed a life-cycle verification and entered the stage of practical application and batch production; high-temperature and medium-load stationary parts, such as a combustion chamber flame tube and an inner and outer lining are undergoing a life-cycle verification and are expected to enter the practical application stage; while high-temperature and high-load rotating parts such as a turbine
rotor and turbine blade are still in the exploratory research stage [3, 4].

The composite materials interface refers to the area with significant changes in chemical composition between the matrix and the reinforcing phase. The interface properties between the fibre and the matrix play an important role on the mechanical behaviour of fibre-reinforced CMCs [5, 6]. The interphase transfers the load between the fibre and the matrix after interface debonding, protects the fibre against an oxidative environment, and deflects the matrix cracking along the interface for the nonlinear behaviour of the CMCs [7, 8, 9, 10, 11, 12, 13]. When the fibre/matrix interface is strongly bonded, the fibre cannot play the role of load carrying, and the fibre-reinforced CMC fractures become brittle; and when the fibre/matrix interface is weakly bonded, the energy dissipation mechanisms of the interface debonding and fibre pullout are not obvious, which limits the nonlinear behaviour of the CMC. When the interface bonding between the fibre and the matrix is suitable, the reinforcing fibre can transfer the load and debonding at a high applied stress, which deflects the matrix cracking and increases the energy dissipation. The tensile and fatigue loading/unloading behaviour is affected by the interface properties (i.e., interface shear stress and interface debonding energy) between the fibre and the matrix [14, 15, 16, 17, 18, 19, 20, 21]. Under tensile loading, the tensile stress-strain curves of fibre-reinforced CMCs can be divided into four different stages, i.e., (1) the linear-elastic stage, and the strain increases in direct proportion to the applied stress; (2) the damage stage with the matrix cracking and fibre debonding occurring at the interface, which makes the CMCs appear to have the characteristics of a pseudo-plastic fracture and high toughness [22]; (3) the damage
stage with the saturation of the matrix cracking and the complete fibre debonding at the interface [23, 24]; and (4) gradually, the fibre fracture stage [25, 26]. The tensile properties including the elastic modulus, proportional limit stress, tensile strength, and fracture strain can be obtained through tensile tests, and used for the design of the CMC components [27]. Ahn et al. [28] investigated the interface deflection/penetration criterion at the fibre and the matrix interface using the energy release rate. Carrere et al. [29] investigated the matrix cracking deflection at the interphase of a mini-SiC/SiC composite with a Pyrocarbon (PyC) interphase. The deflection of the matrix cracking depends on the interface bond strength and interphase type. Sauder et al. [30] investigated the tensile and cyclic loading/unloading behaviour of a mini-SiC/SiC composite with a different interphase. The interphase thickness and the fibre surface roughness affect the tensile nonlinear and fracture strain of the mini-CMCs due to the different interface debonding conditions. Yu et al. [31] investigated the effect of the SiC coating thickness on the mechanical behaviour of a SiC/SiC composite. The flexural strength of a SiC/SiC composite initially increased with the SiC coating thickness and reached a peak value, and then decreased rapidly, however, the bending modulus increased with the SiC coating thickness. Kabel et al. [32] investigated the relationship between the the PyC interphase properties and the debonding shear strength of a SiC/SiC composite. Pryce and Smith [33] investigated the quasi-static tensile behaviour of a unidirectional and cross-ply SiC/CAS composite at room temperature and monitored the evolution of the matrix cracking. Based on the shear-lag model and experimental matrix cracking density, the tensile stress-strain curve of the unidirectional
SiC/CAS composite was predicted [34]. Chateau et al. [35] investigated the damage evolution of a unidirectional SiC/SiC mini-composite using a 1D probabilistic model. The interfacial parameters are obtained by fitting the experimental characterisation. Morscher et al. [36] investigated the stress-dependent matrix cracking evolution of a 2D woven SiC/SiC composite, and the relationship for the stress-dependent matrix cracking could be related to the stress in the load-bearing CVI SiC matrix. Li et al. [37] developed a micromechanical model to predict the tensile behaviour of a unidirectional C/SiC composite, and established the relationship between the tensile curve and the internal damage mechanisms of the matrix cracking, interface debonding, and fibre failure. Li et al. [38] investigated the cyclic loading/unloading tensile behaviour of a unidirectional C/SiC composite at room temperature. The effect of the fibre failure on mechanical hysteresis loops was considered. Li [39, 40] investigated the cyclic loading/unloading hysteresis loops of a cross-ply SiC/MAS composite under the out-of-phase thermomechanical fatigue loading. The relationships between the phase angle, hysteresis loops, loading sequence, and environment temperature were established. The mini-composites (unidirectional composites containing a single bundle of fibres) were used to study the nonlinear behaviour of CMCs for different damage mechanisms [41, 42, 43]. The effect of the interface properties on the frictional heating [44, 45, 46], fatigue damage evolution [47, 48, 49, 50, 51, 52, 53, 54], and electrical resistance [55, 56] of unidirectional SiC/CAS, 2D SiC/SiC composites were investigated. The interface properties affect the tensile and fatigue behaviour of the fibre-reinforced CMCs, especially the nonlinear tensile behaviour and the mechanical
hysteresis loops. It is necessary to perform the investigations on the effect of the interface properties on the tensile and fatigue behaviour of the fibre-reinforced CMCs, in order to establish the relationships between the interface properties and the mechanical behaviour of the CMCs.

In this paper, the nonlinear behaviour of mini-CMCs for different interface properties subjected to tensile and fatigue loading/unloading is investigated. The nonlinear constitutive models for the tensile and fatigue loading/unloading hysteresis loops are developed considering the different damage mechanisms and interface properties. The effect of the interface properties on the nonlinear strain and fracture strain under tensile loading, fatigue loading/unloading hysteresis loops area, strain, and modulus is discussed. The experimental tensile strain and fatigue loading/unloading hysteresis loops of the Hi-Nicalon™ and Tyranno™ SiC/SiC mini-composites with different interphase properties are predicted.

2. Materials and experimental procedures

The Hi-Nicalon™ S (Nippon Carbon Co. Ltd., Takauchi, Japan) and Tyranno SA3 (UBE Industries, Tokyo, Japan) tows reinforced SiC matrix (SiC/SiC) mini-composites were fabricated using the chemical vapour infiltration (CVI) method. The SiC tows were coated with a single layer of pyrocarbon (PyC) with a thickness of 30 or 150 nm, and a multilayer of PyC and SiC with a thickness of 150 nm. The tensile and cyclic loading/unloading experimental results were performed and obtained per Sauder et al. [30]. The tensile and cyclic loading/unloading tests were performed at room temperature
at a constant strain rate of 50μm/min. The deformations of the mini-composite were measured using two-parallel linear-variable differential transformer extensometers.

3. Theoretical analysis

In the present analysis, the constitutive models for the tensile and fatigue loading/unloading of the mini-CMCs have been developed considering the multiple damage mechanisms and interface properties. The stochastic matrix cracking model, fracture mechanics approach, and Global Load Sharing (GLS) criterion are used to determine the space between the matrix cracking, fibre debonding length, fibre failure probability and intact fibre stress, respectively. Under fatigue loading/unloading, the repeated sliding at the interface is taken into consideration for the mechanical hysteresis loops.

3.1 Tensile constitutive model

Under tensile loading, the nonlinear aspect of the CMCs is caused by the different damage mechanisms. In order to establish the nonlinear tensile constitutive model of the CMCs, the micro stress field of the fibre, matrix and the interface along the fibre direction for the multiple damage status should be analysed. When damage occurs, a unit cell is extracted from the mini-CMCs, as shown in Fig. 1. The shear-lag model can be used to analyse the micro stress field of the damaged composite [57]. The fibre axial stress distribution can be determined using Eq. (1) for the different damage regions [37].
\[
\sigma_i(x) = \begin{cases} 
\Phi - \frac{2\tau_i}{r_i} x, & x \in [0, \ell_i] \\
\sigma_{fo} + \left( \Phi - \sigma_{fo} - 2\frac{l_d}{r_i} \right) \exp \left( -\rho \frac{x-l_d}{r_i} \right), & x \in [\ell_i, \ell_c/2] 
\end{cases}
\]  

where \( r_i \) denotes the fibre radius; \( l_d \) denotes the interface debonding length and can be determined using the fracture mechanics approach [37]; \( \ell_c \) denotes the matrix crack spacing and can be determined using the stochastic matrix cracking model [37]; \( \sigma_{fo} \) denotes the fibre axial stress in the interface bonding region; \( \rho \) denotes the shear-lag model parameter; \( \tau_i \) denotes the interface shear stress; and \( \Phi \) denotes the intact fibre stress.

\[
l_d = \frac{r_i}{2} \left( \frac{V_mE_m}{V_mE_c} \frac{1}{\sigma} + 1 \right) - \sqrt{\left( \frac{r_i}{2\rho} \right)^2 + \frac{r_iV_mE_mE_c}{E_c\tau_i^2} \zeta_d} \quad (2)
\]

\[
l_c = l_{sat} \left\{ 1 - \exp \left[ -\left( \frac{\sigma_m}{\sigma_R} \right)^m \right] \right\}^{-1} \quad (3)
\]

where \( V_m \) denotes the matrix volume; \( E_m \) denotes the matrix elastic modulus; \( E_c \) denotes the composite elastic modulus; \( \zeta_d \) denotes the interface debonding energy; \( l_{sat} \) denotes the saturation matrix crack spacing; \( \sigma_m \) denotes the matrix stress; \( \sigma_R \) denotes the matrix cracking characteristic stress; and \( m \) denotes the matrix Weibull modulus.

Considering multiple damage mechanisms, the nonlinear constitutive relationship of the mini-CMCs can be determined using the following equation. [37]

\[
E_c = \Phi \frac{2l_d}{E_i} - \frac{\tau_i}{E_i r_i^2} + \frac{2\sigma_{fo}}{E_i/2 - l_d} - \frac{1}{\rho E_i} \frac{2n}{l_c^2} \left( \Phi - \sigma_{fo} - 2\frac{l_d}{r_i} \right) \times \left[ \exp \left( -\rho \frac{l_c^2 - l_d}{r_i} \right) - 1 \right] - (\alpha_c - \alpha_c) \Delta T 
\]

where \( \alpha_i \) and \( \alpha_c \) denote the fibre and the composite thermal expansion coefficient,
respectively; and $\Delta T$ denotes the temperature difference between the testing and fabricated temperature.

### 3.2 Fatigue loading/unloading constitutive model

Under fatigue loading/unloading, the interface damage status affects the mechanical hysteresis loops. The fatigue loading/unloading constitutive model can be divided into two cases, i.e., (1) the partial debonding at the fibre interface; and (2) the complete debonding at the fibre interface.

For the partial debonding at the fibre interface, the fatigue loading/unloading constitutive relationship can be determined using the following equations. [38]

\[
\varepsilon_{\text{unloading}} = \frac{\Phi_U}{E_I} + 4 \frac{\tau_0}{E_t} \frac{y^2}{r_i l_c} - \frac{\tau_1}{E_t} \frac{(2y-l_d)(2y+l_d-l_c)}{r_i l_c} - (\alpha_c - \alpha_t) \Delta T
\]

(5)

\[
\varepsilon_{\text{reloading}} = \frac{\Phi_R}{E_I} - 4 \frac{\tau_0}{E_t} \frac{z^2}{r_i l_c} + 4 \frac{\tau_1}{E_t} \frac{(y-2z)^2}{r_i l_c} + 2 \frac{\tau_1}{E_t} \frac{(l_d-2y+2z)(l_d+2y-2z-l_c)}{r_i l_c} - (\alpha_c - \alpha_t) \Delta T
\]

(6)

where $\varepsilon_{\text{unloading}}$ denotes the unloading strain; and $\varepsilon_{\text{reloading}}$ denotes the reloading strain; $\Phi_U$ and $\Phi_R$ denote the intact fibres stress upon unloading and reloading, respectively; and $y$ and $z$ denote the interface counter-slip length and new-slip length, respectively.

For the complete debonding at the fibre interface, the fatigue loading/unloading constitutive relationship can be determined using the following equations. [38]

\[
\varepsilon_{\text{unloading}} = \frac{\Phi_U}{E_I} + 4 \frac{\tau_0}{E_t} \frac{y^2}{r_i l_c} - 2 \frac{\tau_1}{E_t} \frac{(2y-l_c/2)^2}{r_i l_c} - (\alpha_c - \alpha_t) \Delta T
\]

(7)

\[
\varepsilon_{\text{reloading}} = \frac{\Phi_R}{E_I} - 4 \frac{\tau_0}{E_t} \frac{z^2}{r_i l_c} + 4 \frac{\tau_1}{E_t} \frac{(y-2z)^2}{r_i l_c} - 2 \frac{\tau_1}{E_t} \frac{(l_c/2-2y+2z)^2}{r_i l_c} - (\alpha_c - \alpha_t) \Delta T
\]

(8)
4. Results and discussion

The interface properties affect the nonlinear mechanical behaviour of the CMCs, especially subjected to the tensile and fatigue loading/unloading conditions. For the CMCs, the interface properties can be characterised using two parameters of the interface shear stress and the interface debonding energy. In this section, the effects of the interface properties on the nonlinear behaviour of the mini-CMCs subjected to the tensile and fatigue loading/unloading are analysed. The Hi-Nicalon™ Type S SiC/SiC mini-composite was used for the case analysis, and the material properties are given by [30]: \( V_f = 44\% \), \( E_f = 372 \) GPa, \( E_m = 550 \) GPa, \( r_f = 6.5 \) μm, \( \alpha_f = 4.5 \times 10^{-6}/°C \), \( \alpha_m = 4.6 \times 10^{-6}/°C \), \( \Delta T = -1000°C \), \( \zeta_d = 0.1 \) J/m², \( \tau_i = 20 \) MPa.

4.1 Effect of the interface properties on tensile nonlinear strain

Li and Song [59] developed an approach to estimate the interface shear stress of fibre-reinforced CMCs using hysteresis loops. The range of the interface shear stress of a unidirectional SiC/CAS composite is between \( \tau_i = 10 \) and 30 MPa. The effect of the interface shear stress (i.e., \( \tau_i = 10, 20, \) and 30 MPa) on the tensile nonlinear strain and the interface debonding of the SiC/SiC mini-composite is shown in Fig. 2. When the interface shear stress increases from \( \tau_i = 10 \) MPa to \( \tau_i = 30 \) MPa, the composite strain corresponding to the different damage stages of matrix cracking, interface debonding, and fibre failure decreases, and the composite failure strain decreases, due to the decrease in the interface debonding length. When the interface shear stress increases, the stress transfer between the fibre and the matrix increases, and the resistance to the
interface debonding also increases, leading to a decrease in the interface debonding length and the nonlinear tensile strain [58]. The composite failure strain decreases from $\varepsilon_f=0.58\%$ at $\tau_i=10$ MPa to $\varepsilon_f=0.47\%$ at $\tau_i=30$ MPa. The interface complete debonding stress increases with the interface shear stress. The interface complete stress increases from $\sigma=660$ MPa at $\tau_i=10$ MPa to $\sigma=737$ MPa at $\tau_i=20$ MPa.

Vagaggini et al. [60] investigated the constituent properties of fibre-reinforced CMCs using hysteresis loops. It was found that the value of the interface debonding energy is $\zeta_d=0$ to 5 J/m$^2$. The effect of the interface debonding energy (i.e., $\zeta_d=1$, 2, and 3 J/m$^2$) on the tensile nonlinear strain and the interface debonding of the SiC/SiC mini-composite is shown in Fig. 3. When the interface debonding energy increases from $\zeta_d=1$ J/m$^2$ to $\zeta_d=3$ J/m$^2$, the composite strain corresponding to the different damage stages decreases, and the composite failure strain decreases, due to the decrease in the interface debonding length. When the interface debonding energy increases, the energy needed for the interface debonding increases, leading to a decrease in the interface debonding length, and a decrease in the nonlinear strain. The composite failure strain decreases from $\varepsilon_f=0.53\%$ at $\zeta_d=1$ J/m$^2$ to $\varepsilon_f=0.4\%$ at $\zeta_d=3$ J/m$^2$. At the same applied stress of $\sigma=780$, the interface debonding length decreases from $2l_d/l_c=1.0$ at $\zeta_d=1$ J/m$^2$ to $2l_d/l_c=0.38$ at $\zeta_d=3$ J/m$^2$.

4.2 Effect of the interface properties on the fatigue loading/unloading hysteresis loops

The effect of the interface shear stress (i.e., $\tau_i=10$, 20, and 30 MPa) on the fatigue
loading/unloading hysteresis loops and the interface slip of the SiC/SiC mini-composite under the peak stress of $\sigma_{\text{max}}=500$ MPa is shown in Fig. 4. When the interface shear stress increases from $\tau_i=10$ MPa to $\tau_i=30$ MPa, the fatigue loading/unloading hysteresis loops area, peak and valley strain decrease, and the fatigue loading/unloading hysteresis modulus increases, due to the decrease in the interface slip length. When the interface shear stress increases, the resistance to the interface debonding increases, leading to a decrease in the interface debonding and slip length, and a decrease in the area, peak and valley strain of the fatigue loading/unloading hysteresis loops. Under the peak stress of $\sigma_{\text{max}}=500$ MPa, the interface slip range decreases from 40% of the matrix crack spacing at $\tau_i=10$ MPa to 13% of the matrix crack spacing at $\tau_i=30$ MPa.

The effect of the interface debonding energy (i.e., $\zeta_d=1, 3,$ and $5$ J/m$^2$) on the fatigue loading/unloading hysteresis loops and the interface slip of the SiC/SiC mini-composite under the peak stress of $\sigma_{\text{max}}=800$ MPa is shown in Fig. 5. When the interface debonding energy increases from $\zeta_d=1$ J/m$^2$ to $\zeta_d=5$ J/m$^2$, the fatigue loading/unloading hysteresis loops area, peak and valley strain decrease, and the fatigue hysteresis modulus increases, due to the decrease in the interface slip length. When the interface debonding energy increases, the energy needed for the interface debonding increases, leading to a decrease in the interface debonding length, and a decrease in the area, peak and valley strain of the fatigue loading/unloading hysteresis loops. Under the peak stress of $\sigma_{\text{max}}=800$ MPa, the interface slip range decreases from 38% of the matrix crack spacing at $\zeta_d=1$ J/m$^2$ to 20% of the matrix crack spacing at $\zeta_d=5$ J/m$^2$. 
5. Experimental comparisons

Sauder et al. [30] performed investigations on the effect of the tensile and cyclic loading/unloading tensile behaviour of SiC/SiC mini-composites with different interface properties. The material properties of the Hi-Nicalon™ and Tyranno™ SiC/SiC mini-composites are listed in Table 1. The tensile nonlinear strain, matrix cracking evolution, fatigue loading/unloading hysteresis loops, and the interface debonding and slip of Hi-Nicalon™ and Tyranno™ SiC/SiC minicomposites are predicted considering the different interface properties.

5.1 Tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Type A Hi-Nicalon™ Type S SiC/SiC mini-composite

The experimental and predicted tensile nonlinear strain, matrix cracking evolution, fatigue loading/unloading hysteresis loops, and interface debonding and sliding of the Type A Hi-Nicalon™ Type S SiC/SiC mini-composite are shown in Fig. 6-9 and Table 2.

Under tensile loading, the tensile stress-strain curve of the SiC/SiC mini-composite is predicted using the tensile constitutive model of Eq. (4), which exhibits obvious nonlinear behaviour between the first matrix cracking stress of about $\sigma_{mc}=400$ MPa and the saturation matrix cracking stress of about $\sigma_{sat}=800$ MPa, and the composite tensile strength is about $\sigma_{UTS}=940$ MPa with the failure strain of $\varepsilon_f=0.64\%$, as shown in Fig. 6(a). The matrix cracking density is predicted using Eq. (3) and increases from $\lambda_{mc}=0.24$/mm at $\sigma_{mc}=400$ MPa to the saturation matrix cracking density of $\lambda_{sat}=2.85$/mm
at \( \sigma_{\text{sat}} = 800 \) MPa, as shown in Fig. 6(b).

Under fatigue loading/unloading, the interface partially debonds (i.e., \( 2l_d/l_c = 0.56 \)), and the fibre partially slides relative to the matrix in the interface debonding region (i.e., \( y(z)/l_d = 0.86 \)) under the peak stress of \( \sigma_{\text{max}} = 500 \) MPa, and the interface debonding length is determined using Eq. (2), and the fatigue loading/unloading hysteresis loop is predicted by Eq. (5) and (6), as shown in Fig. 7; and under the peak stress of \( \sigma_{\text{max}} = 800 \), and 850 MPa, the fibre/matrix interface completely debonds (i.e., \( 2l_d/l_c = 1 \)), and the fibre completely slides relative to the matrix at the matrix crack spacing (i.e., \( y(z)/l_d = 1 \)), and the interface debonding length is determined using Eq. (2), and the fatigue loading/unloading hysteresis loop is predicted by Eq. (7) and (8), as shown in Fig. 8 and 9.

### 5.2 Tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Type B Hi-Nicalon\textsuperscript{TM} Type S SiC/SiC mini-composite

For the Type B Hi-Nicalon\textsuperscript{TM} Type S SiC/SiC mini-composite, the experimental and predicted tensile nonlinear strain, and matrix cracking evolution are shown in Fig. 10 and Table 3. The tensile stress-strain curve is predicted using the tensile constitutive model of Eq. (4), which exhibits obvious nonlinear behaviour between the first matrix cracking stress of \( \sigma_{\text{mc}} = 350 \) MPa and the saturation matrix cracking stress of \( \sigma_{\text{sat}} = 780 \) MPa, and the composite tensile strength is about \( \sigma_{\text{UTS}} = 878 \) MPa with the failure strain of \( \varepsilon_f = 0.54\% \). The matrix cracking density is predicted using Eq. (3) and increases from \( \lambda_{\text{mc}} = 1.1/\text{mm} \) at \( \sigma_{\text{mc}} = 600 \) MPa to \( \lambda_{\text{sat}} = 4.5/\text{mm} \) at \( \sigma_{\text{sat}} = 875 \) MPa.
The experimental and predicted fatigue loading/unloading hysteresis loops and interface slip under the peak stresses of $\sigma_{\text{max}}=632$, 669, 724, 778, 837, and 878 MPa are shown in Fig. 11-16 and Table 3. Upon unloading and reloading, the interface partially debonds and the fibre partially slides relative to the matrix under the peak stresses of $\sigma_{\text{max}}=632$ and 669 MPa, and the interface debonding length is determined using Eq. (2), and the fatigue loading/unloading hysteresis loop is predicted by Eq. (5) and (6); the interface completely debonds and the fibre partially slides relative to the matrix under the peak stresses of $\sigma_{\text{max}}=724$ and 778 MPa, and the interface debonding length is determined using Eq. (2), and the fatigue loading/unloading hysteresis loop is predicted by Eq. (7) and (8); and the interface completely debonds and the fibre completely slides relative to the matrix under the peak stresses of $\sigma_{\text{max}}=837$ and 878 MPa, and the interface debonding length is determined using Eq. (2), and the fatigue loading/unloading hysteresis loop is predicted by Eq. (7) and (8).

5.3 Tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Type C Hi-Nicalon™ Type S SiC/SiC mini-composite

For the Type C Hi-Nicalon™ Type S SiC/SiC mini-composite, the tensile stress-strain curve is predicted using the tensile constitutive model of Eq. (4). The nonlinear tensile strain starts from the first matrix cracking stress of $\sigma_{\text{mc}}=300$ MPa, and the composite tensile strength is about $\sigma_{\text{UTS}}=860$ MPa with the failure strain of $\varepsilon_f=0.62\%$. The matrix cracking density is predicted using Eq. (3) and increases from $\lambda_{\text{mc}}=0.02$/mm at $\sigma_{\text{mc}}=300$ MPa to $\lambda=5.8$/mm at $\sigma_{\text{sat}}=850$ MPa, as shown in Fig. 17 and
Table 4. When the fatigue peak stress increases from $\sigma_{\text{max}}=485$ MPa to $\sigma_{\text{max}}=743$ MPa, the interface completely debonds and the fibre sliding range increases from a partial slip (i.e., $y(z)/l_d<1$) to a complete slip (i.e., $y(z)/l_d=1$), and the interface debonding length is determined using Eq. (2), and the fatigue loading/unloading hysteresis loop is predicted by Eq. (5) and (6), as shown in Fig. 18-21 and Table 4.

5.4 Tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Tyranno™ SA3 SiC/SiC mini-composite

For the Tyranno™ SA3 SiC/SiC mini-composite, the tensile nonlinear strain is predicted using the tensile constitutive model of Eq. (4) and evolves from the first matrix cracking stress of $\sigma_{\text{mc}}=600$ MPa to the saturation matrix cracking stress of $\sigma_{\text{sat}}=1000$ MPa, and the corresponding matrix cracking density is predicted using Eq. (3) and evolves from $\lambda_{\text{mc}}=1.6$/mm to $\lambda_{\text{sat}}=25$/mm, and the composite tensile strength is about $\sigma_{\text{UTS}}=1116$ MPa with the failure strain of $\varepsilon_f=0.58\%$, as shown in Fig. 22 and Table 5.

Under the peak stresses of $\sigma_{\text{max}}=884$, 933, and 985 MPa, the interface debonds partially (i.e., $2l_d/l_c=0.27$, 0.45, and 0.63), and the fibre slides partially relative to the matrix (i.e., $y(z)/l_d=0.83$, 0.78, and 0.73), and the interface debonding length is determined using Eq. (2), and the fatigue loading/unloading hysteresis loop is predicted by Eq. (5) and (6), as shown in Fig. 23-25 and Table 5.

6. Conclusion

In this paper, the tensile nonlinear strain and the fatigue loading/unloading hysteresis loops of the long-fibre-reinforced unidirectional mini-CMCs were
investigated for different interface properties. The nonlinear tensile constitutive model and the fatigue loading/unloading constitutive model were developed considering different damage mechanisms. The relationships between the interface properties, tensile nonlinear strain, and fatigue loading/unloading hysteresis loops were established. The effects of the interface properties on the tensile damage and the fracture, fatigue loading/unloading hysteresis loops area, strain, and modulus were analysed. When the interface shear stress or interface debonding energy increases, the interface debonding length and slip length decrease, leading to a decrease in the tensile damage and failure strain, and the area, peak and residual strain of the fatigue loading/unloading hysteresis loops, and an increase in the fatigue hysteresis modulus. The tensile damage and fracture, and the fatigue loading/unloading hysteresis loops of the Hi-Nicalon™ and Tyranno™ SiC/SiC mini-composites with the different interface properties were analysed. The experimental matrix cracking evolution, tensile nonlinear strain, and fatigue loading/unloading hysteresis loops were predicted, and related to the interface debonding and slip condition. In the present analysis, there are significant differences between the modelling and experimental data of the matrix cracking density evolution curves. However, the experimental matrix cracking evolution is monitored using the the acoustic emission method, which may take the fibre/matrix interface debonding and the fibre failure signals of the matrix cracking into account, leading to the differences between the experimental and predicted results.
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Nomenclature

\( r_f \)  
 fibre radius

\( l_d \)  
 interface debonding length

\( l_c \)  
 matrix crack spacing

\( \tau_i \)  
 interface shear stress

\( \Phi \)  
 intact fibre stress

\( \rho \)  
 shear-lag model parameter

\( \zeta_d \)  
 interface debonding energy

\( l_{sat} \)  
 saturation matrix crack spacing

\( \sigma_{fo} \)  
 fibre axial stress in the bonding region

\( \sigma_m \)  
 matrix stress

\( \sigma_R \)  
 matrix cracking characteristic stress

\( m \)  
 matrix Weibull modulus

\( \alpha_f \)  
 fibre thermal expansional coefficient

\( \alpha_c \)  
 composite thermal expansional coefficient

\( \Delta T \)  
 temperature difference between testing and fabricated temperature

\( y \)  
 unloading interface counter slip length

\( z \)  
 reloading interface new slip length

\( E_f \)  
 fibre elastic modulus

\( E_m \)  
 matrix elastic modulus

\( E_c \)  
 composite elastic modulus
\( \Phi_U \) intact fibres stress upon unloading

\( \Phi_R \) intact fibres stress upon reloading

\( \varepsilon_{\text{unloading}} \) unloading strain

\( \varepsilon_{\text{reloading}} \) reloading strain

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Table 3. The tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Type B Hi-Nicalon™ Type S SiC/SiC mini-composite.

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Table 5. The tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Tyranno™ SA3 SiC/SiC mini-composite.

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Table 1. The material properties of the unidirectional SiC/SiC mini-composites.

<table>
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<tr>
<th>Items</th>
<th>Type A Hi-Nicalon™ S SiC/SiC</th>
<th>Type B Hi-Nicalon™ S SiC/SiC</th>
<th>Type C Hi-Nicalon™ S SiC/SiC</th>
<th>Tyranno™ SA3 SiC/SiC</th>
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Table 2. The tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Type A Hi-Nicalon™ Type S SiC/SiC mini-composite.

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<th>$\sigma_{\text{UTS}}$/MPa</th>
<th>$\varepsilon_{\text{f}}$/%</th>
<th>$\lambda_{\text{mat}}/(\text{mm}^{-1})$</th>
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Table 3. The tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Type B Hi-Nicalon™ Type S SiC/SiC mini-composite.

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<td>$\sigma_{\text{sat}}$/MPa</td>
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| Items   | $\sigma_{\text{max}}$/MPa | $2l_d/l_c$ | $y(z)/l_d$ | $\sigma_{\text{tr_fu}}$/MPa | $\sigma_{\text{tr_fr}}$/MPa |
| Value   | 699     | 0.6     | 0.84      | 878     | 1.0      | 1.0      |

| Items   | $\sigma_{\text{max}}$/MPa | $2l_d/l_c$ | $y(z)/l_d$ | $\sigma_{\text{tr_fu}}$/MPa | $\sigma_{\text{tr_fr}}$/MPa |
| Value   | 724     | 1.0     | 0.55      | 837     | 1.0      | 1.0      |

| Items   | $\sigma_{\text{max}}$/MPa | $2l_d/l_c$ | $y(z)/l_d$ | $\sigma_{\text{tr_fu}}$/MPa | $\sigma_{\text{tr_fr}}$/MPa |
| Value   | 778     | 1.0     | 0.79      | 878     | 1.0      | 1.0      |

| Items   | $\sigma_{\text{max}}$/MPa | $2l_d/l_c$ | $y(z)/l_d$ | $\sigma_{\text{tr_fu}}$/MPa | $\sigma_{\text{tr_fr}}$/MPa |
| Value   | 837     | 1.0     | 1.0       | 41.8    | 795.2    |

| Items   | $\sigma_{\text{max}}$/MPa | $2l_d/l_c$ | $y(z)/l_d$ | $\sigma_{\text{tr_fu}}$/MPa | $\sigma_{\text{tr_fr}}$/MPa |
| Value   | 878     | 1.0     | 1.0       | 175.6   | 702.4    |
Table 4. The tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Type C Hi-Nicalon™ Type S SiC/SiC mini-composite.

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<th>Items</th>
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<th>σ_{UTS}/MPa</th>
<th>ε_{f}/%</th>
<th>λ_{max}/(mm⁻¹)</th>
<th>λ_{sat}/(mm⁻¹)</th>
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Table 5. The tensile nonlinear strain and fatigue loading/unloading hysteresis loops of the Tyranno™ SA3 SiC/SiC mini-composite.

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<th>$\sigma_{\text{sat}}$/MPa</th>
<th>$\sigma_{\text{UTS}}$/MPa</th>
<th>$\varepsilon_f$/%</th>
<th>$\lambda_{\text{max}}$/(mm$^{-1}$)</th>
<th>$\lambda_{\text{sat}}$/(mm$^{-1}$)</th>
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Fig. 5. The effect of the interface debonding energy on (a) the fatigue loading/unloading hysteresis loops; and, (d) the interface slip lengths versus the applied stress curves of the Hi-Nicalon\textsuperscript{TM} Type S SiC/SiC mini-composite.
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