COMPUTATION OF REGIONAL STRESS TENSOR FROM SMALL SCALE TECTONIC DATA

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Abstract

Striations measured in a given faulted area are used for the determination of the stress tensor. Two methods for solving this inverse problem are presented. They were tested through an application to synthetic and actual data. The first test proved the ability to distinguish several tectonic phases. The results of the second test are in good agreement with geological interpretation.

1. Introduction

In the recent time great attention is paid to the determination of the stress state of rock environment. One of the possible approaches is based on the processing of angular measurements of striations on fault planes. Let us suppose that a certain tectonic phase is characterized by a single homogeneous stress tensor that fully determines the direction and sense of the movement on the already existing fault planes. Under this assumption, it is possible to determine the deviatoric part of the stress tensor (Etchecopar et al., 1981).
When the region is subject to the action of one tectonic phase, we speak about a monophased problem, in the case of more phases - about a multiphased problem.

If we denote the unknown stress tensor by $T'$, the unit normal to the fault plane by $n$, and the unit displacement vector on the fault plane by $s$, then the stress vector

$$\sigma = T'n$$

(1)

acting upon this plane has the normal and tangential component (Fig. 1):

$$\sigma_n = nT'n$$

$$\sigma_s = sT'n$$

(2)

Fig. 1. Fault plane, striations and acting stresses. (n - unit normal to the fault plane, s - unit striae on the fault plane, expressing oriented direction of relative motion of the blocks, Tn - stress vector acting on fault plane with normal component (nTn)n and shear component (sTns)).

Płaszczyzna uskoku, rysej i naprężeń (n - wektor jednostkowy prostopadły do płaszczyzny uskoku, s - jednostkowa rysa na płaszczyźnie uskoku wskazująca kierunek względnego ruchu bloków, Tn - wektor naprażeń, działający na płaszczyznę uskoku, o składowej prostopadłej (nTn)n i poprzecznej (sTns)).

The following relation holds for the stress vector acting upon this plane

$$T'n = (nT'n)n + (sT'n)s$$

(3a)

The agreement of the resulting stress sense with the sense of the movement on the fault is expressed by the condition

$$sT'n = 0$$

(3b)

It is advantageous to use three independent angles, $d, p, \omega$, which we obtain from the field measurement (Fig. 2), to the determination of the fault plane normal and of the displacement vector on it. If other angles are measured, it is possible to convert them to the three above-mentioned ones. An example of the conversion is given in the Appendix.

If the $T'$ tensor fulfills condition (3), then all tensors $T$ fulfill it too,

$$T = k_1T' + k_2I$$

(4)

Fig. 2. Fault plane with field measurements. (x, y, z - coordinate system; d - azimuth of dip of fault plane; p - dip of fault plane; $\omega$ - pitch of striae i.e. angle between the horizontal direction of fault plane $h$ and striae $s$; $\delta$ - azimuth of dip of striations (see the Appendix)); $e$ - dip of striations (see the Appendix)).

Płaszczyzna uskoku z zaznaczonymi kątami pochodzący z pomiarów. (x, y, z - układ współrzędnych, $p$ - nachylenie płaszczyzny uskoku, $\omega$ - nachylenie rysy, t.j., kat między kierunkiem poziomym płaszczyzny uskoku, $h$, i rysa $s$; $\delta$ - azymut nachylenia rys (patrz Appendix); $e$ - nachylenie rys (patrz Appendix)).

where $I$ is the isotropic tensor $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $k_1, k_2$ are arbitrary constants; one may prove this by substituting (4) into (3a, b). It is expedient to select from all tensors that fulfill (3) the only one that fulfills further conditions. It will be further demanded that

$$T_{11} + T_{22} + T_{33} = 0$$

(5)

and

$$T_{11}^2 + T_{22}^2 + T_{33}^2 = 3/2.$$  

(6)

By introducing these two conditions, the number of independent parameters decreases from 6 (6 independent components of the $T'$ tensor) to 4, which will be denoted $\alpha, \beta, \gamma, \psi$. It is possible to introduce them, e.g., by the following parametrization of the $T$ tensor:

$$T = \begin{bmatrix} \cos \psi & \alpha & \gamma \\ \alpha & \cos(\psi + 2\pi) & \beta \\ \gamma & \beta & \cos(\psi + 4\pi) \end{bmatrix}$$
that fulfills conditions (5) and (6). All tensors that fulfill (3) and (4) can be obtained from the T tensor by means of relationship (4).

To illustrate the T tensor it is expedient to find its principal stresses $\sigma_1$, $\sigma_2$, $\sigma_3$. Each of the axes is described by azimuth and dip angle, so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Further the parameter $R = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}$, $0 \leq R \leq 1$, is introduced to express the character of T. It is possible to show that for $R = 0$ tensor T has the form of CLVD (compensated linear vector dipole) in axis 1, and for $R = 1$ the T tensor has the form of CLVD in axis 3.

The T tensor fulfilling condition (3) can be found by solving the inverse problem for parameters $\alpha$, $\beta$, $\gamma$, $\psi$. This problem is nonlinear due to the introduction of the supplementary condition (6). In the following section two different ways of solution are presented.

2. Modified Angelier-Tarantola method (ANGE program)

This paragraph is based on the work of Angelier et al. (1982). By a simple modification of relationship (3) the condition is obtained that the searched tensor must fulfill:

$$sTn = (Tn)(Tn) - (nTn)^2.$$  \hspace{1cm} (7)

To determine T conforming to this condition the generalized least squares method is applied, as presented in the paper of Tarantola and Valette (1982). This generalization is based on the idea that the searched tensor parameters and data are considered to be equivalent. In other words, the data are the parameters the values of which are known with a better accuracy than the values of the real parameters. This method is based on the fact that for the given set of angular data denoted $(d_k, p_k, \omega_k)$, where $k = 1...N$ (N - number of measured fault planes), there does not exist such a tensor that would satisfy exactly equation (7) on every fault plane. Therefore, together with the tensor parameters $\psi$, $\alpha$, $\beta$, $\gamma$, a set of "revised" data $(d_k^R, p_k^R, \omega_k^R)$ is searched, that conform to equation (7) on every fault with the requested accuracy. Because an infinitely great number of these revised data exist for each stress tensor, the selection is limited by the condition

$$\sum_{k=1}^{N} \left( \frac{d_k - d_k^R}{\sigma_0} \right)^2 + \left( \frac{p_k - p_k^R}{\sigma_0} \right)^2 + \left( \frac{\omega_k - \omega_k^R}{\sigma_0} \right)^2 + \left( \frac{\psi - \psi_k^R}{\sigma_0} \right)^2 + \left( \frac{\alpha - \alpha_k^R}{\sigma_0} \right)^2 + \left( \frac{\beta - \beta_k^R}{\sigma_0} \right)^2 + \left( \frac{\gamma - \gamma_k^R}{\sigma_0} \right)^2 = \text{min.}$$  \hspace{1cm} (8)

where $\sigma_0$ - assumed standard deviations, $\psi_0$, $\alpha_0$, $\beta_0$, $\gamma_0$ - primary estimate of the T tensor parameters.

In accordance with the suggested considerations, the common vector of unknowns is introduced

$$\pi = (\psi, \alpha, \beta, \gamma, (d, p, \omega))^T,$$

which is the solution of the system of equations

$$F^T(\pi) = 0,$$  \hspace{1cm} (10)

where

$$F^k = \begin{pmatrix} (T_k^n)^T - (n_k^n)^T \end{pmatrix},$$

$$n_k^n = (\sin \psi_k^n \sin \beta_k^n \cos \alpha_k^n \sin p_k^n \cos \omega_k^n \cos \gamma_k^n, \sin \omega_k^n \cos \beta_k^n \cos \alpha_k^n \sin p_k^n \cos \gamma_k^n, -\sin \psi_k^n \cos \beta_k^n \cos \alpha_k^n \sin p_k^n \cos \gamma_k^n, \sin \omega_k^n \cos \beta_k^n \cos \alpha_k^n \sin p_k^n \sin \gamma_k^n, \sin \omega_k^n \sin \beta_k^n \cos \alpha_k^n \sin p_k^n \cos \gamma_k^n),$$

$$s_k^n = (-\sin \omega_k^n \cos \beta_k^n \sin \alpha_k^n + \cos \omega_k^n \cos \beta_k^n \cos \gamma_k^n, -\sin \omega_k^n \cos \beta_k^n \cos \alpha_k^n \sin p_k^n - \cos \omega_k^n \cos \beta_k^n \cos \gamma_k^n, \sin \omega_k^n \sin \beta_k^n \cos \alpha_k^n \sin p_k^n - \cos \omega_k^n \sin \beta_k^n \cos \gamma_k^n, \sin \omega_k^n \sin \beta_k^n \sin \alpha_k^n + \cos \omega_k^n \sin \beta_k^n \cos \gamma_k^n).$$

In this way the inverse problem is defined, the solution of which can be found according to the algorithm (Tarantola and Valette, 1982):

$$\pi_{n+1} = \pi_n + C_0^{-1} [F_n(\pi_n - \pi_0) - f(\pi_n)],$$  \hspace{1cm} (11)

where $C_0$ is a covariant matrix of $\pi_0$:

$$F_n = \begin{pmatrix} \delta F_n^1 \\ \delta F_n^2 \\ \vdots \\ \delta F_n^n \end{pmatrix},$$

$$S_{n+1} = \sum_{i=1}^{N} |f(\pi_{n+1})|.\hspace{1cm} (13)$$

If this quantity attains a lower value than the preselected limit the iterations are stopped. Angelier et al. (1982) represent the opinion that a value of $10^{-5}$ is low enough. However, it can be seen that the value should be chosen depending on the number of data and their quality; in some cases, the solution does not change markedly even when choosing $10^{-3}$.

The presented algorithm of calculation is suitable only for the data pertinent to one tectonic phase. In the case of multiphased data, the resulting solution is sort of a mean value of individual phases. Therefore, the algorithm was modified in such a way that the processing of the multiphased data became possible. Within the calculation, the value is determined for every point

$$\max \left( |d - d_0| + |p - p_0| + |m - m_0| \right)^k,$$  \hspace{1cm} (14)
are searching for the parameters \( x_i \), \( i = 1, \ldots, m \), that have to conform in a certain optimal way to the system of nonlinear equations
\[
y_j = f_j(x_1, \ldots, x_m)
\]
(17)
By optimal solution of (17) the parameters \( x_i \) assume such values that the residual sum
\[
R = \sum_{j=1}^{n} \frac{(y_j - f_j(x_1, \ldots, x_m))^2}{\sigma_j^2}
\]
(18)
is minimum.

The search for optimal parameters proceeds in two stages. In the first one, the space of parameters is scanned with a constant step. According to criterion (18), the best estimate of parameters is selected to serve further as a starting model. In the second phase the \( x_i \) parameters are searched by an iterative procedure. Let us suppose that in the \( k \)-iteration the parameters \( x_i^k \) are obtained. The \( (k+1) \)-approximation is determined as
\[
x_i^{k+1} = x_i^k + \alpha^k \Delta x_i^k
\]
(19)
where \( \alpha^k \) is multiplication factor which fluctuates about 1, and \( \Delta x_i^k \) is determined by solving the linear equation system
\[
\sum_{i,j=1}^{n} B_{ij} B_{11} \Delta x_i^k = \sum_{i=1}^{m} B_{ij} (y_i - f_i)
\]
\[
\sigma_i^2
\]
(20)
where \( B_{ij} = \frac{\partial y_i}{\partial x_j} \) (Matsuura and Hasegawa, 1987).

The derivatives matrix \( B_{ij} \) is calculated numerically in the OBRAC program. The three-point derivative with variable step is used. This step is determined from the parameter increment in the preceding iteration.

When the matrix
\[
B_{ij} = \frac{\partial^2 y_i}{\partial x_j^2}
\]
is an ill-conditioned one, then a complication arises. In such cases the updating factor is usually introduced (Marquardt, 1970). In the OBRAC program, another procedure was chosen:

If we consider the dispersion of \( y_i \) to be \( \sigma_i \), then the dispersion of the right side in (20) is \( S_j = \sum_{i=1}^{m} B_{ij} \). The \( S_j^2 \) variance, calculated in this way enters a special subprogram for the solution of linear equations, which calculates not only the solution of the system but also the error of this solution.
The calculation is interrupted when for some \( \Delta x \), solution its error is higher then \( 2\Delta x \), and the calculation is repeated in such a manner that \( x \) is considered to be constant in the given iteration.

The OBRAC program was applied to the stress tensor calculation. The quantities \( \alpha, \beta, \gamma, \phi \) were selected to be parameters in the same way as in the preceding method. The measured angles \( d, p, \omega \) serve as data.

The direct problem is described by the following formulae:

\[
T = T(\alpha, \beta, \gamma, \phi), \quad a = Tn - (nT)n, \quad \omega = \arccos(\frac{\mathbf{a} \cdot \mathbf{h}}{||\mathbf{a}|| \cdot ||\mathbf{h}||})
\]

where: \( n \) - fault plane normal, \( h \) - directional vector of the fault plane intersection line with the horizontal plane.

It is possible to set the data into the program with known orientation (normal or reversed fault) or with unknown orientation, then it is necessary to add \( 180^\circ \) to the \( \omega \) angle, if it better satisfies the data.

The program enables the solution of the multiphased problem. Within the computation, the data for which the difference in the measured and calculated \( \omega \)-angles is greater than \( 30^\circ \) are temporarily excluded in every iteration. In this way, the calculation of one phase is not influenced by the presence of another phase if the difference between them is big enough.

After the calculations are finished, all data with the residuum lower then \( 15^\circ \) are selected. These data are excluded and the calculation is repeated for the rest of the data.

4. Application to the two-phased artificial data

The correct operation of both programs was tested on the set of artificial data. Twenty-four fault planes were selected with the orientation approximately isotropically distributed into all directions. Two directions of the slip on fault were calculated on each fault plane that correspond to two different tensors. The angle values were changed at random maximum by \( 1^\circ \). From these data the UMEL set including 48 data points was formed, to which both programs were applied. The results of this analysis are illustrated in Fig. 3 and Table I.

The OBRAC program differentiated both phases correctly and calculated both tensors. The only basic difference between the expected and found solution is the fact that 28 data points were assigned to the first phase, among which 4 data points actually belonged to the second phase. The whole computation for both phases lasted 74 s.

![Fig. 3. Application to the two-phased artificial data. Schmidt's projection of the lower hemisphere; Left: Fault planes (arcs) and corresponding striations (circles with bars showing the movement direction). Right: Tensors computed by both methods. (Circles denote the first phase, squares denote the second one. The largest, the intermediate and the smallest marks are the axes of the maximum, intermediate and minimum stress, respectively).](image)

**Table I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Theory</th>
<th>ANGE program</th>
<th>OBRAC program</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_1 )</td>
<td>0</td>
<td>-0.017</td>
<td>-0.040</td>
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<tr>
<td>( \alpha_1 )</td>
<td>1</td>
<td>1.005</td>
<td>1.030</td>
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<tr>
<td>( \beta_1 )</td>
<td>0</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0</td>
<td>0.002</td>
<td>0.010</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>0.229</td>
<td>0.191</td>
<td>0.214</td>
</tr>
<tr>
<td>( \psi_2 )</td>
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<td>1.014</td>
<td>1.010</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>-0.4</td>
<td>-0.338</td>
<td>-0.402</td>
</tr>
<tr>
<td>( \beta_2 )</td>
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<td>0.999</td>
<td>0.987</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
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<td>0.984</td>
<td>1.006</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>0.597</td>
<td>0.585</td>
<td>0.593</td>
</tr>
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</table>
The ANGE program differentiated the two phases and calculated tensors too. The differences in the determination of $\psi$, $\alpha$, $\beta$, $\gamma$ are of the order of $10^{-2}$. However, the possibility to exclude the data that were not appurtenant to the searched phase is limited; 12 data points appurtenant to the second phase were assigned to the first phase. The computation lasted several minutes.

5. Calculation for real data

Both methods were applied in the study of the tectogenesis of the middle part of the Bohemian Cretaceous basin (Coubal, 1990) in the SE part of the Lusatian fault near Turnov. The basic structure was formed by the Kobrov flexure that originated from the several-hundred-meters upheaval of the NE block in the first documented post-Cretaceous tectonic phase. On this block the fracture tectonic phenomena were superposed in four phases (Coubal, 1990).

Further upheaval caused the breaking of the flexure shoulder by a fault along which the NE block was abruptly slid towards SW. The bedding in the underlying block was reoriented by the sliding into vertical, in places into overturned setting.

During the increase of the strata bending, the slipages along the layer planes occurred; in the compressed bending core the shears of the S blocks towards N originated on the planes near the bedding (Fig. 4, planes b).

Fig. 4. An example of mutual dislocation of geological and tectonic elements on the site in which the data set S8 was obtained.

Fig. 5. Application to the real data (40 fault planes in the Bohemian Cretaceous basin, SE part of the Lusatian fault). Crosses denote the only phase determined by program ANGE, circles and squares denote the two phases determined by program OBRAC. Other symbols as in Fig. 3.

Zastosowanie obu programów do danych rzeczywistych (40 płaszczyzn uskoków w Czeskim Zagłębiu Kredowym, w południowo-wschodniej części uskoku Lusatian). Krzyżyki oznaczają jedyną fazę wyznaczoną programem ANGE, kółka i kwadraty oznaczają dwie fazy wyznaczone programem OBRAC. Pozostałe symbole jak na rys. 3.

6. Discussion

It follows from the processing of artificial data that the OBRAC program is better fit for separating different tectonic phases in a multiphased data set. Moreover, the application of the OBRAC program is not limited by the number of measured points like the ANGE program, in which the computation time
Table II
Results of an analysis of the S8 set of real data
Wyniki analizy rzeczywistych danych zbioru S8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ANGE program</th>
<th>OBRAC program</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1$</td>
<td>1.008</td>
<td>-1.047</td>
</tr>
<tr>
<td>$\alpha_1$</td>
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<tr>
<td>$\beta_1$</td>
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<td>$\gamma_1$</td>
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<td>$R_1$</td>
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<tr>
<td>$\psi_2$</td>
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<tr>
<td>$\alpha_2$</td>
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<td>0.104</td>
</tr>
<tr>
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</tr>
<tr>
<td>$R_2$</td>
<td>-</td>
<td>0.878</td>
</tr>
</tbody>
</table>

and the memory request increase markedly with the increasing number of data.

Using artificial data it was shown that both methods give the same results if there is a low measurement error and the differences in principal stress axes of individual phases are big enough.

The results of the application to real data show certain disagreement. The OBRAC program determined two tectonic phases (Fig. 5) with the stress tensor orientation corresponding to the geologically estimated data (Fig. 4). The presence of both tectonic phases is demonstrated by the mutual dislocation of the geological and tectonic elements. The effects of both estimated phases of the Saxon tectogenesis were described even in other places of the central part of the Bohemian Cretaceous basin (Couhal, 1990) and also in other basins of the Alpine foreland (Bergerat, 1987).

On the other hand, the ANGE program determined only one phase, the stress tensor orientation of which is identical with the second geologically proved phase. All data but one were assigned to this phase; the revised data, however, show a very high deviation (14) from the original data $(d_0, P_0, w_0)$k. This demonstrates the fact that the program "revised" the data so that they might conform to the parameters of the one phase tensor. The resulting solution corresponds to the revised data set different from the original data. This is documented in Fig. 6, which illustrates the resulting stress tensor fact. It is possible to see that the OBRAC program found here only one phase too, the stress tensor of which is congruent with the tensor found by ANGE program. The mentioned ability "to revise the data", that can be advantageous in processing the monophased data, seems to be not suitable for the processing of multiphase data.

Fig. 6. Application to the "revised" real data S8T (see text). Explanations — see Fig. 3.

Zastosowanie obu programów do "poprawionych" danych rzeczywistych S8T (patrz tekst). Objaśnienia — patrz rys. 3.

7. Conclusion

Two programs were developed for the computation of the regional stress tensor from small scale tectonic data; they solve the problem in different ways. The ANGE program uses the method according to Angler et al. (1982) complemented with the possibility to process the multiphased data. The OBRAC program was developed on the base of the weighted least squares method. The case when the matrices used for the computation are ill-conditioned is solved by reducing the number of parameters in the corresponding iteration. The program enables to process multiphased data, that in the case of measurement with a low error on isotropically spaced fault planes and sufficiently distant axes of individual phases both programs differentiate with confidence the individual phases and determine correctly the tensor parameters. The computation by means of the OBRAC program is several times faster and thanks to its low memory request it can process greater data sets. The assignment of data to individual phases is not reliable in either program; it may happen that some data belonging to a less pronounced phase be assigned to the most pronounced one.
The computation for real data proved the usability of both programs in practice. The OBRAC program is more sensitive to differentiate the individual phases than the ANGE program. The obtained results are in agreement with the geological conclusions.

The accuracy and reliability of the obtained results are not absolute, especially when more tectonic phases are present; nevertheless, the processing of small scale tectonic data by the described methods, notably the OBRAC program, can contribute significantly to the knowledge of the tectonic evolution of the region, particularly in combination with further geological examination.

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References


OBliczanie Regionalnego Tensora Naprzeń
NA PODSTAWIE DANYCH MIKROTEKTONICZNYCH

S t r e s z c z e n i e

APPENDIX

In geological praxis, the pair of angles $\delta$, $\varepsilon$ and movement orientation (normal, reverse) is usually taken instead of the angle $\omega$ (Fig. 5). The unit displacement vector $s$ is then of the form

$$s = (-\sin \delta \cos \varepsilon, -\cos \delta \cos \varepsilon, \sin \varepsilon)$$

for normal fault

and

$$s = (\sin \delta \cos \varepsilon, \cos \delta \cos \varepsilon, -\sin \varepsilon)$$

for reverse fault.

The angle $\omega$ is then determined as

$$\omega = \arccos(h)$$

for normal fault

and

$$\omega = 2\pi - \arccos(h)$$

for reverse fault,

where $h = (\cos \delta, \sin \delta, 0)$ is the unit horizontal direction of the fault plane.

Since more angles than needed are measured, one is able to estimate the measurement error. The displacement vector $s$ and the normal vector $n$ which are determined independently should be orthogonal. The former is determined by angles $\delta$, $\varepsilon$, the latter by angles $d$ and $p$. The measurement error is then expressed as

$$\Delta = \arccos(sn) - \pi/2$$

This error is used as a criterion for selection of data. If $\Delta > 10^\circ$, the measurement point is excluded.